

General Certificate of Education
January 2005
Advanced Level Examination



MATHEMATICS (SPECIFICATION A)
Unit Pure 4

MAP4

Friday 21 January 2005 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The cubic equation

$$x^3 + px^2 + qx + 30 = 0,$$

where p and q are real, has a root $\alpha = 1 + 2i$.

(a) Write down the other non-real root, β , of the equation. (1 mark)

(b) Find:

(i) $\alpha\beta$; (1 mark)

(ii) the third root, γ , of the equation. (2 marks)

(c) Hence, or otherwise, find the values of p and q . (3 marks)

2 (a) Show that

$$r^2(r+1)^2 - (r-1)^2r^2 = 4r^3. \quad (2 \text{ marks})$$

(b) Use the method of differences to find the value of $\sum_{r=50}^{100} r^3$. (4 marks)

3 (a) Express $1 + i$ in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. (2 marks)

(b) Show that

$$(1 + i)^{21} - (1 - i)^{21} = ki,$$

where k is an integer to be found. (5 marks)

4 (a) On the **same** Argand diagram, sketch the loci of points satisfying:

(i) $|z + 3 + i| = 5$; (2 marks)

(ii) $\arg(z + 3) = -\frac{3}{4}\pi$. (2 marks)

(b) (i) From your sketch, explain why there is only one complex number satisfying both equations. (1 mark)

(ii) Verify that this complex number is $-7 - 4i$. (4 marks)

- 5 (a) (i) Use the definitions $\cosh x = \frac{1}{2}(e^x + e^{-x})$ and $\sinh x = \frac{1}{2}(e^x - e^{-x})$ to show that the equation

$$p \cosh x + q \sinh x = r,$$

may be written as

$$(p + q)e^{2x} - 2re^x + (p - q) = 0. \quad (3 \text{ marks})$$

- (ii) Given that p , q and r are positive and that

$$p^2 = q^2 + r^2,$$

show that the equation

$$p \cosh x + q \sinh x = r$$

has just one solution. (5 marks)

- (b) Find the solution of the equation

$$13 \cosh x + 5 \sinh x = 12,$$

giving your answer in the form $\ln k$, where k is a rational number. (2 marks)

- 6 (a) Given that

$$f(n) = 4 \times 7^n + 3 \times 5^n + 5,$$

show that

$$f(n + 1) - f(n) = 24 \times 7^n + 12 \times 5^n. \quad (4 \text{ marks})$$

- (b) Prove by induction that

$$4 \times 7^n + 3 \times 5^n + 5$$

is a multiple of 12 for all $n \geq 1$. (4 marks)

7 (a) Show that

$$\frac{d}{dx} \left(\sinh^{-1} x + x\sqrt{1+x^2} \right) = 2\sqrt{1+x^2}. \quad (4 \text{ marks})$$

(b) The arc of the curve $y = e^x$ between the points where $x = \ln\left(\frac{3}{4}\right)$ and $x = \ln\left(\frac{4}{3}\right)$ is rotated through 2π radians about the x -axis.

(i) Show that S , the surface area generated, is given by

$$S = 2\pi \int_{\ln\left(\frac{3}{4}\right)}^{\ln\left(\frac{4}{3}\right)} e^x \sqrt{1+e^{2x}} \, dx. \quad (2 \text{ marks})$$

(ii) Use the substitution $u = e^x$ to evaluate S , giving your answer in the form

$$\pi(a + \ln b),$$

where a and b are rational numbers.

(7 marks)

END OF QUESTIONS