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General Certificate of Education

Mathematics 6300
Specification A

MAP4 Pure 4

Mark Scheme

2005 examination - June series

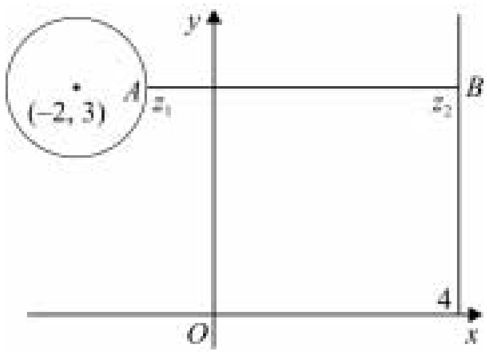
Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

MAP4

Q	Solution	Marks	Total	Comments
1	Attempt to either: make the coefficient of z or w the same or: substitute for either z or w	M1		
	Correctly done	A1		
	$w(1+\mathrm{i})=2$ or $(1+\mathrm{i})z=1+3\mathrm{i}$	A1F		OE
	$w=\frac{2}{1+\mathrm{i}}\times\frac{1-\mathrm{i}}{1-\mathrm{i}}$	M1		OE for z
	$=1-\mathrm{i}$	A1F		
	$z=2+\mathrm{i}$	B1F	6	
				Alternative: $z=a+ib$, $w=c+id$ gives: $-b+2c=1$ $a+2d=0$ M1A1 $a-c+d=0$ $b-d-c=1$ A1 solving M1 w A1F z A1F
	Total		**6**	
2(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\sinh x$	B1		
	$s=\int_{-1}^{1}\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$ used	M1		
	$=\int_{-1}^{1}\sqrt{1+\sinh^2 x}\,\mathrm{d}x$	A1F		ft sign error
	$=\int_{-1}^{1}\cosh x\,\mathrm{d}x$	A1	4	AG
(b)	$\int\cosh x\,\mathrm{d}x=\sinh x$	M1		
	$\sinh 1-\sinh(-1)$	A1		OE
	$\mathrm{e}-\frac{1}{\mathrm{e}}$	A1F	3	
	Total		**7**	

MAP4 (cont)

Q	Solution	Marks	Total	Comments
3(a)				
	circle centre correct	B1		
	radius correct	B1		must not intersect axes
	half line through (4, 0)	B1		
	perpendicular to x-axis	B1	4	
(b)	Required distance is AB	M1		
	$AB = 5$	A1F	2	
	Total		**6**	
4(a)	$\sum \alpha = 0$	B1	1	
(b)(i)	Adequate reason	E1	1	needs to be clear, eg α is a root of the cubic
(ii)	$\sum \alpha^3 = 11\sum \alpha + 450$	M1A1		
	$= 450$	A1	3	AG
(c)	$\beta = -3 - 4\text{i},\ \gamma = 6$	B1B1F	2	B1 could be ft, eg from $\sum \alpha = 11$
				Alternative for (d):
(d)	$(-3+4\text{i})^3 + (-3-4\text{i})^3 + 6^3 = 450$	M1 A1F		attempt to expand $(3 - 4\text{i})^3$ and $(3 + 4\text{i})^3$ correct expansions
	$(3-4\text{i})^3 + (3+4\text{i})^3 = -234$	A1	3	AG
	Total		**10**	

MAP4 (cont)

Q	Solution	Marks	Total	Comments
5	Assume result true for $n = k$			
	Then $u_{k+1} = \frac{1}{2}\left(\left(\frac{1}{2}\right)^{k-1} + k - 2 + k\right)$	M1A1		
	$= \left(\frac{1}{2}\right)^{k} + k - 1$	A1		
	$= \left(\frac{1}{2}\right)^{k} + (k+1) - 2$	A1		clearly seen
	$u_1 = 0$ since $\left(\frac{1}{2}\right)^{0} + 1 - 2 = 0$	B1		
	$P_k \Rightarrow P_{k+1}$ and $P_1 = 0$	E1		must have earned **all** previous marks
	Total		**6**	

MAP4 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Correct general shape	B1		must not intersect asymptotes
	Asymptotes clearly shown	B1	2	
(b)(i)	$\tanh^2 x = \dfrac{\sinh^2 x}{\cosh^2 x} = \dfrac{\cosh^2 x - 1}{\cosh^2 x}$	M1		use of both formulae
	$= 1 - \operatorname{sech}^2 x$	A1	2	AG
(ii)	$\dfrac{d}{dx}\dfrac{\sinh x}{\cosh x} = \dfrac{\cosh^2 x - \sinh^2 x}{\cosh^2 x}$	M1A1		
	$= \operatorname{sech}^2 x$	A1	3	AG
(c)(i)	$\int_0^1 \tanh^2 x\,dx = \int_0^1 \left(1 - \operatorname{sech}^2 x\right)dx$	M1		
	$= [x - \tanh x]_0^1$	A1		
	$= 1 - \tanh 1$	A1	3	AG
(ii)	Put $u = \tanh x$ $\quad \dfrac{du}{dx} = \operatorname{sech}^2 x$	M1		
	$\int_0^1 \tanh^2 x \operatorname{sech}^2 x\,dx = \int_{x=0}^{x=1} u^2\,du$	A1		
	$= \left[\dfrac{u^3}{3}\right]_0^{\tanh 1}$	A1		
	$= \dfrac{1}{3}\tanh^3 1$	A1	4	
(iii)	$\int_0^1 \tanh^4 x\,dx = \int_0^1 \tanh^2 x\,dx$			
	$-\int_0^1 \tanh^2 x \operatorname{sech}^2 x\,dx$	M1		
	$= 1 - \tanh 1 - \dfrac{1}{3}\tanh^3 1$	A1F	2	
	Total		**16**	

MAP4 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\sqrt{3}+\mathrm{i}=2\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}}$	B1B1		
	$2-2\mathrm{i}=\sqrt{8}\mathrm{e}^{-\frac{1}{4}\pi\mathrm{i}}$	B1B1	4	
(b)	$z^3=\dfrac{2\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}}}{\sqrt{8}\mathrm{e}^{-\frac{1}{4}\pi\mathrm{i}}}$	M1		
	$=\dfrac{1}{\sqrt{2}}\mathrm{e}^{\frac{5\pi\mathrm{i}}{12}}$	A1F		
	$z=\dfrac{1}{2^{\frac{1}{6}}}\mathrm{e}^{\frac{5\pi\mathrm{i}}{36}+\frac{2k\pi\mathrm{i}}{3}}$	M1		If M0 lost, allow B1 for $\dfrac{1}{2^{\frac{1}{6}}}\mathrm{e}^{\frac{5\pi\mathrm{i}}{36}}$ OE
	$=\dfrac{1}{2^{\frac{1}{6}}}\mathrm{e}^{\frac{5\pi\mathrm{i}}{36}}$, $\dfrac{1}{2^{\frac{1}{6}}}\mathrm{e}^{\frac{29\pi\mathrm{i}}{36}}$ and $\dfrac{1}{2^{\frac{1}{6}}}\mathrm{e}^{\frac{-19\pi\mathrm{i}}{36}}$	A2,1,0F	5	Accept 0.891 for $\dfrac{1}{2^{\frac{1}{6}}}$ or any equivalent expression
	Total		**9**	
	TOTAL		**60**	