Mark scheme January 2004

GCE

Mathematics A

Unit MAP4

Key to mark scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m mark and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
– *x* EE		Deduct *x* marks for each error
NMS		No method shown
PI		Perhaps implied
c		Candidate

Abbreviations used in marking

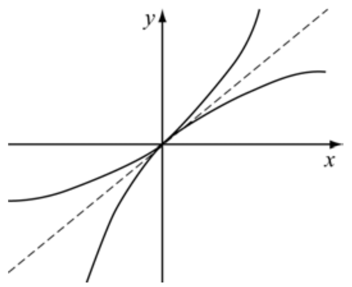
MC – *x*	deducted *x* marks for miscopy
MR – *x*	deducted *x* marks for misread
ISW	ignored subsequent working
BOD	gave benefit of doubt
WR	work replaced by candidate

Application of mark scheme

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Q	Solution	Marks	Total	Comments
1 (a)(i)	$(3+i)^2 = 8+6i$	B1	1	
(ii)	$(2+4i)(3+i) = 2+14i$	B1	1	
(b)(i)	$8+6i-(2+14i)+8i-6=0$	M1A1	2	
(ii)	$z_1+z_2 = 2+4i$	B1	1	
(iii)	coefficients of quadratic not real	E1	1	
(iv)	$z_2 = -1+3i$	B1F	1	
(c)(i)	Points plotted	B1F	1	
(ii)	$\lvert z_1 \rvert = \sqrt{10} = \lvert z_2 \rvert$	M1A1	2	
(iii)	$\arg \dfrac{z_2}{z_1} = \arg\ z_2 - \arg z_1$	M1A1		Any correct method M1 Applied A1
	$= \dfrac{1}{2}\pi$ (Pythagoras, rotation etc)	A1	3	Allow use of decimals
	Total		**13**	
2	$(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})^7 = \cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}$	B1		
	$(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3})^5$			
	$= \cos\frac{5\pi}{3} - i\sin\frac{5\pi}{3}$	B1		Or
	Expansion of $= (\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6})(\cos\frac{5\pi}{3} - i\sin\frac{5\pi}{3})$	M1		$\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$ M1A1 $-\frac{\sqrt{3}}{4} - \frac{3}{4}i - \frac{1}{4}i + \frac{\sqrt{3}}{4}$ A1 $-i$ A1
	$= \cos\left(\frac{7\pi}{6} - \frac{5\pi}{3}\right) + i\sin\left(\frac{7\pi}{6} - \frac{5\pi}{3}\right)$	A1		Allow sign error
	$= \cos(-\frac{\pi}{2}) + i\sin(-\frac{\pi}{2})$	A1		
	$= -i$	A1	6	AG
	Total		**6**	

Q	Solution	Marks	Total	Comments
3 (a)	$\mathrm{f}(n+1)-\mathrm{f}(n)=(n+3)^3-n^3$ $=n^3+3n^2\times 3+3n\times 9+27-n^3$ $=9n^2+27n+27$	M1A1 A1 A1F	4	or attempt at $\mathrm{f}(n+1)-\mathrm{f}(n)$ M1 $3n^3+18n^2+42n+36$ A1 $3n^3+9n^2+15n+9$ A1 result A1
(b)	Assume result true for $n=k$ ie $\mathrm{f}(k)=\mathrm{M}(9)$ Then $\mathrm{f}(k+1)=\mathrm{f}(k)+\mathrm{M}(9)$ $=\mathrm{M}(9)+\mathrm{M}(9)=\mathrm{M}(9)$ But $\mathrm{f}(1)=1^3+2^3+3^3=36=\mathrm{M}(9)$ P_1 true and $P_k \Rightarrow P_{k+1}$ $\therefore$ true by induction	M1A1 A1 B1 E1	5	Must be clear for this A1 Only if correct or almost correct
	Total		**9**	
4 (a)	$\sinh y = x$ $\cosh y \dfrac{\mathrm{d}y}{\mathrm{d}x}=1$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{\cosh y}=\dfrac{1}{\sqrt{x^2+1}}$	M1 A1 A1	3	or $\dfrac{\mathrm{d}}{\mathrm{d}x}\ln\left(x+\sqrt{x^2+1}\right)$ M1 OE correctly differentiated A1 Result A1 AG
(b)(i)	$y=\sinh x,\ \dfrac{\mathrm{d}y}{\mathrm{d}x}=\cosh x=1$ when $x=0$ $y=\sinh^{-1}x,\ \dfrac{\mathrm{d}y}{\mathrm{d}x}=1$ when $x=0$	B1 B1	2	
(ii)	for all x, $\cosh x \geq 1$ for all x, $\sqrt{x^2+1}\geq 1 \therefore \dfrac{1}{\sqrt{x^2+1}}\leq 1$	B1 B2,1,0	3	
(iii)	Sketch of $y=\sinh x$ Sketch of $y=\sinh^{-1}x$  	B1 B1	2	CAO curves must **not** cut for these marks
	Total		**10**	

Q	Solution	Marks	Total	Comments
5 (a)	$\frac{dy}{dx} = \frac{-2x}{1-x^2}$	B1, B1		B1 each numerator and denominator
	$1+\left(\frac{dy}{dx}\right)^2 = 1+\frac{4x^2}{(1-x^2)^2}$	M1		
	$= \frac{(1-x^2)^2+4x^2}{(1-x^2)^2}$	A1F		
	$= \frac{1-2x^2+x^4+4x^2}{(1-x^2)^2}$	A1		CAO
	$= \left(\frac{1+x^2}{1-x^2}\right)^2$	A1	6	
(b)	arc length $= \int_0^p \left(\frac{1+x^2}{1-x^2}\right)dx$	M1		
	$= \int_0^p \left(\frac{2}{1-x^2}-1\right)dx$	A1		
	$\left[2\tanh^{-1}x - x\right]_0^p$	A1F		ft if hyperbolic
	$= 2\tanh^{-1}p - p$	A1	4	AG
	Total		**10**	

Q	Solution	Marks	Total	Comments
6 (a)(i)	$\left(2e^{\frac{\pi i}{4}}\right)^4 = 16e^{\pi i} = -16$	B1	1	
	$z = 2e^{\left(\frac{\pi i}{4}+\frac{2k\pi i}{4}\right)}$	M1		
	$k=0,\ z=2e^{\frac{\pi i}{4}}$			
	other roots, $z = 2e^{-\pi i/4}$, $z = 2e^{\pm 3\pi i/4}$	A2,1,0	3	Allow if quoted correctly Deduct A1 for answers outside range indicated
(iii)	Argand diagram: $r = 2$ Properly spaced	B1 B1	2	CAO except for $r = 2$
(b)(i)	$\left(z-2e^{\frac{\pi i}{4}}\right)\left(z-2e^{-\frac{\pi i}{4}}\right)$			
	$= z^2 - 2\left(e^{\frac{\pi i}{4}} + e^{-\frac{\pi i}{4}}\right)z + 4e^{\frac{\pi i}{4}}e^{-\frac{\pi i}{4}}$	M1		
	$= z^2 - 2\times 2\cos\frac{\pi}{4}z + 4$	A1		Must see some working for this A1
	$= z^2 - 2\sqrt{2}z + 4$	A1	3	AG
(ii)	$(z-2e^{3\pi i/4})(z-2e^{-3\pi i/4})$			
	$= z^2 - 2\times 2\cos\frac{3\pi}{4}z + 4 = z^2 + 2\sqrt{2}z + 4$	M1A1		
	$z^4 + 16 = (z^2 - 2\sqrt{2}z + 4)(z^2 + 2\sqrt{2}z + 4)$	A1	3	If quoted allow B1
	Total		**12**	
	Total		**60**	