

Mark scheme January 2004

GCE

Mathematics A

Unit MAP4

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Key to mark scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m mark and is for	accuracy
В	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
or ft or F		follow through from previous
		incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
-x EE		Deduct x marks for each error
NMS		No method shown
PI		Perhaps implied
c		Candidate

Abbreviations used in marking

MC-x	deducted x marks for miscopy
MR-x	deducted x marks for misread
ISW	ignored subsequent working
BOD	gave benefit of doubt
WR	work replaced by candidate

Application of mark scheme

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Q	Solution	Marks	Total	Comments
1 (a)(i)	$(3+i)^2 = 8+6i$	B1	1	
(ii)	(2+4i)(3+i) = 2+14i	B1	1	
(b)(i)	8 + 6i - (2 + 14i) + 8i - 6 = 0	M1A1	2	
(ii)	$z_1 + z_2 = 2 + 4i$	B1	1	
(iii)	coefficients of quadratic not real	E1	1	
(iv)	$z_2 = -1 + 3i$	B1F	1	
(c)(i)	Points plotted	B1F	1	
(ii)	$\left z_{1}\right = \sqrt{10} = \left z_{2}\right $	M1A1	2	
(iii)	$\arg \frac{z_2}{z_1} = \arg z_2 - \arg z_1$	M1A1		Any correct method M1 Applied A1
	$= \frac{1}{2}\pi$ (Pythagoras, rotation etc)	A1	3	Allow use of decimals
	Total		13	
2	$\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^7 = \cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}$	B1		
	$(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3})^5$			
	$=\cos\frac{5\pi}{3}-\mathrm{i}\sin\frac{5\pi}{3}$	B1		
	Expansion of			Or
	$= (\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6})(\cos\frac{5\pi}{3} - i\sin\frac{5\pi}{3})$	M1		$\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \qquad M1A1$
				$-\frac{\sqrt{3}}{4} - \frac{3}{4}i - \frac{1}{4}i + \frac{\sqrt{3}}{4}$ A1
	$=\cos\left(\frac{7\pi}{6} - \frac{5\pi}{3}\right) + i\sin\left(\frac{7\pi}{6} - \frac{5\pi}{3}\right)$	A1		Allow sign error
	$=\cos(-\frac{\pi}{2})+i\sin(-\frac{\pi}{2})$	A1		
	= -i	A1	6	AG
	Total		6	

3 (a) $f(n+1)-f(n) = (n+3)^3 - n^3$ M1A1 or attempt at $f(n+1)-f(n)$ M1 $= n^3 + 3n^2 \times 3 + 3n \times 9 + 27 - n^3$ A1 $3n^3 + 18n^2 + 42n + 36$ A1 $= 9n^2 + 27n + 27$ A1F A1F A3 $3n^3 + 9n^2 + 15n + 9$ A1 Assume result true for $n = k$ is $f(k) = M(9)$ M1A1 Must be clear for this A1 But $f(1) = 1^2 + 2^2 + 2^3 = 36 = M(9)$ A1 M1A1 Must be clear for this A1 Pi true and $f_k \Rightarrow f_{k+1}$ A1 9 or $\frac{d}{dx} \ln(x + \sqrt{x^2 + 1})$ M1 Cosh $y \frac{dy}{dx} = 1$ A1 A1 OE correctly differentiated A1 A1 A2 A3 A3 A3 A3 A4 A4 A4 A4 A4 A4 (a) Sinh $y = x$ A1 A1 A2 A3 A3 A4 (a) $\frac{dy}{dx} = \frac{1}{\cosh y} = \frac{1}{\sqrt{x^2 + 1}}$ A1 A3 A3 A3 A3 (b)(b) $y = \sinh^1 x$, $\frac{dy}{dx} = \cosh x = 1$ when $x = 0$ B1 2 A3 A3 (iii) for all x , $\sqrt{x^2 + 1} \ge 1$. $\frac{1}{\sqrt{x^2 + 1}} = 1$ B2,1,0 A3	Q	Solution	Marks	Total	Comments
(b) $\frac{1}{2} = \frac{1}{3} + \frac{1}{3} + \frac{1}{2} \times 3 + 3n \times 9 + 27 - n^3}{2} = \frac{1}{9n^2 + 27n + 27}$ A1 $\frac{1}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \times 3 + 3n \times 9 + 27 - n^3}{2} = \frac{1}{9n^2 + 27n + 27}$ A1 A1F (b) Assume result true for $n = k$ is $f(k) = M(9)$ Then $f(k+1) = f(k) + M(9)$ $\frac{1}{2} = M(9) + M(9) = M(9)$ But $f(1) = 1^3 + 2^3 + 3^3 = 36 = M(9)$ A1 A1 But $f(1) = 1^3 + 2^3 + 3^3 = 36 = M(9)$ A1			MIAI		or attempt at
(b) (b) $= 9n^2 + 27n + 27$ Assume result true for $n = k$ ie $f(k) = M(9)$ $= M(9) + M(9) = M(9)$ But $f(1) = 1^3 + 2^3 + 3^3 = 36 = M(9)$ P_t true and $P_t = P_{k+1}$ \therefore true by induction Total 4 (a) $\frac{dy}{dx} = \frac{1}{\cosh y} = \frac{1}{\sqrt{x^2 + 1}}$ Al Al Al Al Al Al Must be clear for this Al Only if correct or almost correct or $\frac{d}{dx} \ln(x + \sqrt{x^2 + 1})$ MI OE correctly differentiated Al Result Al AG (b)(i) $y = \sinh^{-1}x, \frac{dy}{dx} = \cosh x = 1 \text{ when } x = 0$ Bl $y = \sinh^{-1}x, \frac{dy}{dx} = 1$ Bl for all $x, \cos h x \ge 1$ for all $x, \cos h x \ge 1$ Sketch of $y = \sinh^{-1}x$ Bl Sketch of $y = \sinh^{-1}x$ Bl Sketch of $y = \sinh^{-1}x$ Bl CAO curves must not cut for these marks		1(n+1)-1(n)=(n+3)-n	WITAT		
(b) Assume result true for $n = k$ is $f(k) = M(9)$ Then $f(k+1) = f(k) + M(9)$ But $f(1) = 1^3 + 2^3 + 3^3 = 36 = M(9)$ P_1 true and $P_k \Rightarrow P_{k+1}$ true by induction Total 4 (a) $\sinh y = x$ $\cosh y \frac{dy}{dx} = 1$ Al Al Al Al Al Al Al Al Al Bi Only if correct or almost correct Total OE correctly differentiated Al Result Al Al AG (b)(i) $y = \sinh^{-1}x, \frac{dy}{dx} = \cosh x = 1 \text{ when } x = 0$ Bl $y = \sinh^{-1}x, \frac{dy}{dx} = 1 \text{ when } x = 0$ Bl for all $x, \sqrt{x^2 + 1} \ge 1 \cdot \frac{1}{\sqrt{x^2 + 1}} \le 1$ Sketch of $y = \sinh^{-1}x$ Bl Sketch of $y = \sinh^{-1}x$ Bl Sketch of $y = \sinh^{-1}x$ Bl CAO curves must not cut for these marks		$= n^3 + 3n^2 \times 3 + 3n \times 9 + 27 - n^3$	A1		$3n^3 + 18n^2 + 42n + 36 \qquad A1$
Assume result true for $n = k$ is if $(k) = M(9)$ Then $f(k+1) = f(k) + M(9)$ $= M(9) + M(9) = M(9)$ But $f(1) = 1^3 + 2^3 + 3^3 = 36 = M(9)$ f_1 true and $f_k \Rightarrow f_{k+1}$ \therefore true by induction Total Total Total $\frac{dy}{dx} = \frac{1}{\cosh y} = \frac{1}{\sqrt{x^2 + 1}}$ Al $\frac{dy}{dx} = \cosh x = 1 \text{ when } x = 0$ Bl $y = \sinh^{-1} x, \frac{dy}{dx} = 1 \text{ when } x = 0$ Bl $y = \sinh^{-1} x, \frac{dy}{dx} = 1 \text{ when } x = 0$ Bl $for \text{ all } x, \sqrt{x^2 + 1} \ge 1 \therefore \frac{1}{\sqrt{x^2 + 1}} \le 1$ Sketch of $y = \sinh^{-1} x$ Bl Sketch of $y = \sinh^{-1} x$ Bl CAO curves must not cut for these marks		$=9n^2+27n+27$	A1F	4	$3n^3 + 9n^2 + 15n + 9 $ A1
ie $f(k) = M(9)$ Then $f(k+1) = f(k) + M(9)$ $= M(9) + M(9) = M(9)$ But $f(1) = 1^3 + 2^3 + 3^3 = 36 = M(9)$ P_i true and $P_k \Rightarrow P_{k+1}$ \therefore true by induction Total 4 (a) $\sinh y = x$ $\cosh y \frac{dy}{dx} = 1$ $\cosh y \frac{dy}{dx} = \cosh x = 1 \text{ when } x = 0$ B1 $y = \sinh^{-1} x, \frac{dy}{dx} = 1 \text{ when } x = 0$ B1 $y = \sinh^{-1} x, \frac{dy}{dx} = 1 \text{ when } x = 0$ B1 $\cosh x = \frac{1}{\sqrt{x^2 + 1}} + \frac{1}{\sqrt{x^2 + 1}} $	(b)	Assume result true for $n = k$			result A1
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$					
But $f(1) = 1^3 + 2^3 + 3^3 = 36 = M(9)$ P_1 true and $P_k \Rightarrow P_{k+1}$ \therefore true by induction Total 9 4 (a) $\sinh y = x$ $\cosh y \frac{dy}{dx} = 1$ $\cosh y \frac{dy}{dx} = \cosh x = 1 \text{ when } x = 0$ (ii) $for all \ x, \sqrt{x^2 + 1} \ge 1 \therefore \frac{1}{\sqrt{x^2 + 1}} \le 1$ Sketch of $y = \sinh^{-1}x$ $\Rightarrow x$					
But $1(p) = 1 + 2 + 3 = 36 = M(9)$ P_1 true and $P_k \Rightarrow P_{k+1}$.true by induction Total 9 4 (a) $\sinh y = x$ $\cosh y \frac{dy}{dx} = 1$ Al $\frac{dy}{dx} = \frac{1}{\cosh y} = \frac{1}{\sqrt{x^2 + 1}}$ Al Al (b)(i) $y = \sinh^{-1}x, \frac{dy}{dx} = 1$ when $x = 0$ B1 $y = \sinh^{-1}x, \frac{dy}{dx} = 1$ when $x = 0$ B1 for all x , $\sqrt{x^2 + 1} \ge 1 \therefore \frac{1}{\sqrt{x^2 + 1}} \le 1$ Sketch of $y = \sinh^{-1}x$ B1 B2, 1,0 3 CAO curves must not cut for these marks					Must be also for this A1
Total T					Must be clear for this AT
Total 9 4 (a) $\sinh y = x$ $\cosh y \frac{dy}{dx} = 1$ M1 OE correctly differentiated A1 Result A1 AG (b)(i) $y = \sinh x$, $\frac{dy}{dx} = \cosh x = 1$ when $x = 0$ B1 $y = \sinh^{-1}x$, $\frac{dy}{dx} = 1$ when $x = 0$ B1 for all x , $\sqrt{x^2 + 1} \ge 1$. $\frac{1}{\sqrt{x^2 + 1}} \le 1$ Sketch of $y = \sinh^{-1}x$ B1 Sketch of $y = \sinh^{-1}x$ B1 Sketch of $y = \sinh^{-1}x$ B1 CAO curves must not cut for these marks				5	Only if correct or almost correct
4 (a) $\sinh y = x$ $\cosh y \frac{dy}{dx} = 1$ M1 A1 OE correctly differentiated A1 Result A1 AG (b)(i) $y = \sinh x$, $\frac{dy}{dx} = \cosh x = 1$ when $x = 0$ B1 $y = \sinh^{-1}x$, $\frac{dy}{dx} = 1$ when $x = 0$ B1 for all x , $\sqrt{x^2 + 1} \ge 1$ $\therefore \frac{1}{\sqrt{x^2 + 1}} \le 1$ Sketch of $y = \sinh^{-1}x$ B1 Sketch of $y = \sinh^{-1}x$ B1 B2 CAO curves must not cut for these marks		•		0	
$\cosh y \frac{dy}{dx} = 1$ $\cosh y \frac{dy}{dx} = \frac{1}{\cosh y} = \frac{1}{\sqrt{x^2 + 1}}$ $A1$ 3 AG $(b)(i) y = \sinh x, \frac{dy}{dx} = \cosh x = 1 \text{ when } x = 0$ $y = \sinh^{-1} x, \frac{dy}{dx} = 1 \text{ when } x = 0$ $B1$ $y = \sinh^{-1} x, \frac{dy}{dx} = 1 \text{ when } x = 0$ $B1$ $for all \ x, \cosh x \ge 1$ $for all \ x, \sqrt{x^2 + 1} \ge 1 \therefore \frac{1}{\sqrt{x^2 + 1}} \le 1$ $Sketch \text{ of } y = \sinh^{-1} x$ $B1$ $Sketch \text{ of } y = \sinh^{-1} x$ $B1$ 2 $CAO \text{ curves must not cut for these marks}$	4 (a)		M1	, ,	d ()
OE correctly differentiated A1 Result A1 AG (b)(i) $y = \sinh x$, $\frac{dy}{dx} = \cosh x = 1 \text{ when } x = 0$ B1 $y = \sinh^{-1} x$, $\frac{dy}{dx} = 1 \text{ when } x = 0$ B1 for all x , $\sqrt{x^2 + 1} \ge 1$: $\frac{1}{\sqrt{x^2 + 1}} \le 1$ Sketch of $y = \sinh x$ Sketch of $y = \sinh^{-1} x$ B1 CAO curves must not cut for these marks	7 (a)				or $\frac{d}{dx} \ln(x + \sqrt{x^2 + 1})$ M1
$\frac{dy}{dx} = \frac{1}{\cosh y} = \frac{1}{\sqrt{x^2 + 1}}$ A1 A2 A3 AG Result A1 AG AG (b)(i) $y = \sinh x$, $\frac{dy}{dx} = \cosh x = 1 \text{ when } x = 0$ B1 $y = \sinh^{-1} x, \frac{dy}{dx} = 1 \text{ when } x = 0$ B1 $for all \ x, \ \sqrt{x^2 + 1} \ge 1 \therefore \frac{1}{\sqrt{x^2 + 1}} \le 1$ B2,1,0 B1 Sketch of $y = \sinh^{-1} x$ B1 B1 CAO curves must not cut for these marks		$\cosh y \frac{dy}{dx} = 1$	A1		
$\frac{dy}{dx} = \frac{1}{\cosh y} = \frac{1}{\sqrt{x^2 + 1}}$ Al $y = \sinh x, \frac{dy}{dx} = \cosh x = 1 \text{ when } x = 0$ Bl $y = \sinh^{-1} x, \frac{dy}{dx} = 1 \text{ when } x = 0$ Bl $for \text{ all } x, \cosh x \ge 1$ For all $x, \sqrt{x^2 + 1} \ge 1 \therefore \frac{1}{\sqrt{x^2 + 1}} \le 1$ Sketch of $y = \sinh^{-1} x$ Bl $Sketch \text{ of } y = \sinh^{-1} x$ Bl $Sketch \text{ of } y = \sinh^{-1} x$ CAO curves must not cut for these marks		u.			OE correctly differentiated A1
(b)(i) $y = \sinh x$, $\frac{dy}{dx} = \cosh x = 1 \text{ when } x = 0$ B1 $y = \sinh^{-1} x$, $\frac{dy}{dx} = 1 \text{ when } x = 0$ B1 2 (ii) for all x , $\cos x \ge 1$ B1 B2,1,0 B2,10 B1 B2,10 B1 B2,10 B1 B2,10 B1 B1 B1 B1 CAO curves must not cut for these marks					Result A1
(b)(i) $y = \sinh x$, $\frac{dy}{dx} = \cosh x = 1 \text{ when } x = 0$ B1 $y = \sinh^{-1} x$, $\frac{dy}{dx} = 1 \text{ when } x = 0$ B1 2 (ii) for all x , $\cos x \ge 1$ B1 B2,1,0 B2,10 B1 B2,10 B1 B2,10 B1 B2,10 B1 B1 B1 B1 CAO curves must not cut for these marks		$\frac{dy}{dx} = \frac{1}{\cosh y} = \frac{1}{\sqrt{2}}$	A1	3	AG
$y = \sinh^{-1} x, \frac{dy}{dx} = 1 \text{ when } x = 0$ B1 2 (ii) for all x , $\cos x \ge 1$ for all x , $\sqrt{x^2 + 1} \ge 1$: $\frac{1}{\sqrt{x^2 + 1}} \le 1$ Sketch of $y = \sinh x$ Sketch of $y = \sinh^{-1} x$ B1 2 CAO curves must not cut for these marks		$\sqrt{x^2+1}$			
$y = \sinh^{-1} x, \frac{dy}{dx} = 1 \text{ when } x = 0$ B1 2 (ii) for all x , $\cos x \ge 1$ for all x , $\sqrt{x^2 + 1} \ge 1$: $\frac{1}{\sqrt{x^2 + 1}} \le 1$ Sketch of $y = \sinh x$ Sketch of $y = \sinh^{-1} x$ B1 2 CAO curves must not cut for these marks	(b)(i)	$v = \sinh x$, $\frac{dy}{dx} = \cosh x = 1$ when $x = 0$	B1		
(ii) for all x , $\cosh x \ge 1$ for all x , $\sqrt{x^2 + 1} \ge 1$. $\frac{1}{\sqrt{x^2 + 1}} \le 1$ Sketch of $y = \sinh x$ Sketch of $y = \sinh^{-1}x$ B1 B2,1,0 B1 B1 CAO curves must not cut for these marks		dx			
(ii) for all x , $\cosh x \ge 1$ for all x , $\sqrt{x^2 + 1} \ge 1$. $\frac{1}{\sqrt{x^2 + 1}} \le 1$ Sketch of $y = \sinh x$ Sketch of $y = \sinh^{-1}x$ B1 B2,1,0 B1 B1 CAO curves must not cut for these marks		$v = \sinh^{-1} x$, $\frac{dy}{dx} = 1$ when $x = 0$	B1	2	
for all x , $\sqrt{x^2 + 1} \ge 1$ $\frac{1}{\sqrt{x^2 + 1}} \le 1$ Sketch of $y = \sinh x$ Sketch of $y = \sinh^{-1}x$ B1 B1 CAO curves must not cut for these marks		dx			
(iii) Sketch of $y = \sinh x$ Sketch of $y = \sinh^{-1}x$ B1 2 CAO curves must not cut for these marks	(ii)	for all x , $\cosh x \ge 1$	B1		
(iii) Sketch of $y = \sinh x$ Sketch of $y = \sinh^{-1}x$ B1 2 CAO curves must not cut for these marks		1			
(iii) Sketch of $y = \sinh x$ Sketch of $y = \sinh^{-1}x$ B1 2 CAO curves must not cut for these marks		for all x , $\sqrt{x^2 + 1} \ge 1$: $\frac{1}{\sqrt{x^2 + 1}} \le 1$	B2,1,0	3	
Sketch of $y = \sinh^{-1}x$ B1 CAO curves must not cut for these marks	(iii)				
$\frac{y}{x}$	(111)			2.	CAO curves must not cut for these marks
		Sketch of $y = \sinh^{-x}x$	Di	_	
		y			
Total 10		$\frac{1}{x}$			
Total 10					
		Total		10	

Q	Solution	Marks	Total	Comments
5 (a)	$dx = 1 - x^2$	B1, B1		B1 each numerator and denominator
	$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{4x^2}{\left(1 - x^2\right)^2}$	M1		
	$=\frac{(1-x^2)^2+4x^2}{(1-x^2)^2}$	A1F		
(b)	$=\frac{1-2x^2+x^4+4x^2}{\left(1-x^2\right)^2}$	A1		CAO
	$= \left(\frac{1+x^2}{1-x^2}\right)^2$	A1	6	
	arc length = $\int_{0}^{p} \left(\frac{1+x^2}{1-x^2} \right) dx$	M1		
	$= \int_{0}^{p} \left(\frac{2}{1 - x^2} - 1 \right) \mathrm{d}x$	A1		
	$\left[2\tanh^{-1}x-x\right]_0^p$	A1F		ft if hyperbolic
	$= 2 \tanh^{-1} p - p$	A1	4	AG
	Total		10	

Q	Solution	Marks	Total	Comments
6 (a)(i)	$\left(2e^{\frac{\pi i}{4}}\right)^4 = 16e^{\pi i} = -16$	B1	1	
	$z = 2e^{\left(\frac{\pi i}{4} + \frac{2k\pi i}{4}\right)}$	M1		
	$k=0, z=2e^{\frac{\pi i}{4}}$			
	other roots, $z = 2e^{-\pi i/4}$, $z = 2e^{\pm 3\pi i/4}$	A2,1,0	3	Allow if quoted correctly Deduct A1 for answers outside range indicated
(iii)	Argand diagram: $r = 2$ Properly spaced	B1 B1	2	CAO except for $r = 2$
(b)(i)	$\left(z - 2e^{\frac{\pi i}{4}}\right)\left(z - 2e^{-\frac{\pi i}{4}}\right)$			
	$=z^{2}-2\left(e^{\frac{\pi i}{4}}+e^{-\frac{\pi i}{4}}\right)z+4e^{\frac{\pi i}{4}}e^{-\frac{\pi i}{4}}$	M1		
	$= z^2 - 2 \times 2 \cos \frac{\pi}{4} z + 4$	A1		Must see some working for this A1
	$=z^2-2\sqrt{2}z+4$	A1	3	AG
(ii)	(2-2e)(2-2e)			
	$= z^{2} - 2 \times 2 \cos \frac{3\pi}{4} z + 4 = z^{2} + 2\sqrt{2}z + 4$	M1A1		
	$z^4 + 16 = (z^2 - 2\sqrt{2}z + 4)(z^2 + 2\sqrt{2}z + 4)$	A1	3	If quoted allow B1
	Total		12	
	Total		60	