General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAP3**
Unit Pure 3

Wednesday 23 June 2004 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A curve is given by the parametric equations

$$x = 2t - 1, \quad y = \frac{1}{2t}.$$

(a) Find $\frac{dy}{dx}$ in terms of t. *(2 marks)*

(b) Find the equation of the normal to the curve at the point where $t = 1$. *(4 marks)*

2 (a) Obtain the binomial expansion of $(1 + x)^{\frac{1}{3}}$ as far as the term in x^2. *(2 marks)*

(b) Hence, or otherwise, find the series expansion of $(8 + 4x)^{\frac{1}{3}}$ as far as the term in x^2. *(3 marks)*

3 (a) Express $\frac{30}{(x+4)(7-2x)}$ in the form $\frac{A}{x+4} + \frac{B}{7-2x}$. *(3 marks)*

(b) Hence find

$$\int_0^3 \frac{30}{(x+4)(7-2x)}\,dx,$$

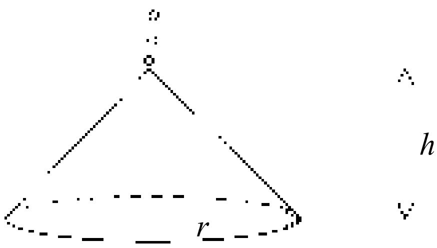
giving your answer in the form $p \ln q$, where p and q are rational numbers. *(5 marks)*

4 A curve is given by the equation $9(y+2)^2 = 5 + 4(x-1)^2$.

(a) Find the coordinates of the two points on the curve where $x = 2$. *(3 marks)*

(b) Find the gradient of the curve at each of these points. *(5 marks)*

5 In a timing device, sand falls through a small hole to form a conical heap. As the cone forms, the height, h cm, remains equal to the base radius, r cm, of the heap.



(a) The volume of the sand after t minutes is V cm^3. Explain why $V = \frac{1}{3}\pi h^3$. *(1 mark)*

(b) The sand falls through the hole at a rate of 3 cm^3 per minute. Find the rate at which the height of the heap is increasing at the instant when $h = 2$. Give your answer to two significant figures. *(3 marks)*

6 The function f is given by $f(x) = \sin\left(2x + \frac{\pi}{6}\right)$.

(a) (i) Find $f'(x)$ and $f''(x)$. *(3 marks)*

(ii) Hence, using the Maclaurin series, show that, for small values of x,

$$f(x) \approx \frac{1}{2} + \sqrt{3}x - x^2.$$ *(2 marks)*

(b) Show that, for small values of x,

$$(1 - \cos x)\sin\left(2x + \frac{\pi}{6}\right) \approx kx^2,$$

where the value of the constant k is to be found. *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

7 Initially there are 2000 fish in a lake. The number of fish, x, at time t months later is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x(1 - kt),$$

where k is a constant.

(a) Solve this differential equation to show that

$$x = 2000\mathrm{e}^{t - \frac{1}{2}kt^2}.$$ *(6 marks)*

(b) After 12 months the number of fish is again 2000. Find the value of k. *(3 marks)*

8 (a) Find the vector equation of the line l_1, which passes through the points $A(3, -1, 2)$ and $B(2, 0, 2)$. *(2 marks)*

(b) The line l_2 has vector equation $\mathbf{r} = \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$.

Show that the lines l_1 and l_2 intersect and find the coordinates of their point of intersection. *(4 marks)*

(c) Show that the point $C(9, 1, -6)$ lies on the line l_2. *(2 marks)*

(d) Find the coordinates of the point D on l_1 such that CD is perpendicular to l_1. *(4 marks)*

END OF QUESTIONS