

General Certificate of Education  
January 2004  
Advanced Level Examination



**MATHEMATICS (SPECIFICATION A)**  
**Unit Pure 3**

**MAP3**

Friday 23 January 2004 Morning Session

**In addition to this paper you will require:**

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

**Information**

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

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1 A curve is given by the parametric equations

$$x = 3t^2, \quad y = 6t.$$

- (a) (i) Find  $\frac{dy}{dx}$  in terms of  $t$ . (2 marks)
- (ii) Find the gradient of the curve at the point where  $t = \frac{1}{2}$ . (1 mark)
- (b) (i) Find the equation of the curve in the form  $x = f(y)$ . (2 marks)
- (ii) Find  $\frac{dx}{dy}$  in terms of  $y$  and hence verify your answer to part (a)(ii). (4 marks)

2 A curve satisfies the differential equation

$$\frac{dy}{dx} = \frac{1}{1+x^3}.$$

Starting at the point  $(1, 0.5)$  on the curve, use a step-by-step method with a step length of 0.25 to estimate the value of  $y$  at  $x = 1.5$ , giving your answer to two decimal places. (5 marks)

3 A microbiologist is studying the growth of populations of simple organisms.

For one such organism, the model proposed is

$$P = 100 - 50e^{-\frac{1}{4}t},$$

where  $P$  is the population after  $t$  minutes.

- (a) Write down:
- (i) the initial value of the population; (1 mark)
- (ii) the value which the population approaches as  $t$  becomes large. (1 mark)
- (b) Find the time at which the population will have a value of 75, giving your answer to two significant figures. (4 marks)

4 (a) Express  $\frac{8 + 3x}{(1 + 3x)(2 - x)}$  in the form  $\frac{A}{1 + 3x} + \frac{B}{2 - x}$ . (3 marks)

(b) Obtain the first three terms in the expansion of  $\frac{1}{1 + 3x}$  in ascending powers of  $x$ . (2 marks)

(c) Show that the first three terms in the expansion of  $\frac{1}{2 - x}$  in ascending powers of  $x$  are  $\frac{1}{2} + \frac{x}{4} + \frac{x^2}{8}$ . (3 marks)

(d) Hence, or otherwise, obtain the first three terms in the expansion of  $\frac{8 + 3x}{(1 + 3x)(2 - x)}$  in ascending powers of  $x$ . (3 marks)

(e) State the range of values of  $x$  for which the expansion in part (d) is valid. (2 marks)

5 (a) (i) The function  $f$  is given by

$$f(x) = e^{-2x}.$$

By differentiation, find  $f'(x)$  and  $f''(x)$ . (2 marks)

(ii) Hence show that the first three terms in the Maclaurin series of  $f(x)$  are

$$1 - 2x + 2x^2. \quad (2 \text{ marks})$$

(b) (i) Using the approximation  $\cos x \approx 1 - \frac{x^2}{2}$ , write down a similar approximation for  $\cos 3x$ . (1 mark)

(ii) Use your results from parts (a)(ii) and (b)(i) to find an approximate solution of the equation

$$e^{-2x} = \cos 3x \quad \text{for } 0 < x < 1. \quad (3 \text{ marks})$$

- 6 The speed  $v \text{ m s}^{-1}$  of a pebble falling through still water after  $t$  seconds can be modelled by the differential equation

$$\frac{dv}{dt} = 10 - 5v.$$

A pebble is placed carefully on the surface of the water at time  $t = 0$  and begins to sink.

- (a) Show that  $t = \frac{1}{5} \ln\left(\frac{2}{2-v}\right)$ . (6 marks)
- (b) Use the model to find the speed of the pebble after 0.5 seconds, giving your answer to two significant figures. (3 marks)

- 7 The points  $A$  and  $B$  have coordinates  $(3, -1, 2)$  and  $(5, 3, -2)$  respectively.

- (a) (i) Find the distance between  $A$  and  $B$ . (2 marks)
- (ii) Find the coordinates of  $M$ , where  $M$  is the mid-point of  $AB$ . (1 mark)

- (b) The point  $C$  has coordinates  $(8, -2, -1)$ .

Show that  $\vec{CM}$  is perpendicular to  $\vec{AB}$ . (2 marks)

- (c) Points  $A$  and  $B$  both lie in a plane  $\Pi$ .

Given that the line  $CM$  is perpendicular to the plane  $\Pi$ , show that the Cartesian equation of  $\Pi$  is

$$4x - 3y - z = 13. \quad (2 \text{ marks})$$

- (d) The line  $CP$  has equation  $\mathbf{r} = \begin{pmatrix} 8 \\ -2 \\ -1 \end{pmatrix} + t \begin{pmatrix} 5 \\ -3 \\ 3 \end{pmatrix}$  and intersects  $\Pi$  at the point  $P$ .

Find the coordinates of the point  $P$ . (3 marks)

**END OF QUESTIONS**