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General Certificate of Education

Mathematics 6300
Specification A

MAP3 Pure 3

Mark Scheme

2005 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

MAP3

Q	Solution	Marks	Total	Comments
1(a)	$1-5x+10x^2-10x^3+5x^4-x^5$	M1		1, 5, 10, 10, 5, 1 (6 terms needed) attempt numerical coefficients
		A1	2	CAO
(b)	$-10\times3^2\times2^3=-720$	M1		$3^5\left(\frac{2}{3}\right)^2\times10$ or $\frac{5\times4\times3}{2\times3}\times3^2\times2^3$ are acceptable
		A1	2	CAO
				SC $1080x^2$ 1/2
	Total		**4**	
2(a)	$\left(\frac{dy}{dx}=\right)\frac{dy}{dt}\frac{dt}{dx}=\frac{4}{-2t}$	M1		use of chain rule if $\frac{dx}{dy}$ attempted, must be clearly stated
		A1	2	ISW where appropriate
				SC eliminate t first: award M1 for correct use of chain rule, A1 for $\frac{-2}{t}$
(b)	when $t=4$, $\frac{dy}{dx}=\frac{-1}{2}$	B1F		for evaluating gradient ft deriv of any function of t **except** $-2t$
	gradient of normal = 2	B1F		ft on gradient; could still be in terms of t
	$y=2x+c$ $t=4$ $x=-14$ $y=16$	M1		use of their (–14, 16) and gradient
	$y=2x+44$	A1F	4	ft on gradient if tangent found, needs to be in form $y=-\frac{1}{2}x+9$
	Total		**6**	
3(a)(i)	$P=20$	B1	1	
(ii)	$P=20e^{1.5}=89.6\approx90$	M1		
		A1	2	89 or 90 (**not** 89.6) 89.6 without working: M1 A0
(b)	$50=e^{\frac{t-6}{4}}$ $\left(\text{or } 1000=20e^{\frac{t-6}{4}}\right)$	M1		or $1001=20e^{\frac{t-6}{4}}$ or $1000.5=20e^{\frac{t-6}{4}}$
	$\ln 50=\frac{t-6}{4}$	M1		taking logs
	$t=21.6$	A1	3	22 acceptable if working shown (otherwise 2/3)
	Total		**6**	

MAP3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	x y step x $\frac{dy}{dx}$ step y			
	1 3 0.5 0.1666.. 0.0833..	M1A1		
	1.5 3.0833 0.5 0.1208.. 0.0604..	M1		
	2 3.1437..	A1		allow premature rounding errors. $3\frac{157}{1092}$
	$x = 3.144$	A1	5	CAO (must have at least 4 dp throughout working)
(b)	$\frac{5-x^2}{25-x^2} = \frac{25-x^2-20}{25-x^2} = 1 - \frac{20}{25-x^2}$	B1	1	AG
(c)	$20 = A(5+x) + B(5-x)$	M1		OE; A and B need to be the right way round
	$x = 5 \quad A = 2, \qquad x = -5 \quad B = 2$	A1	2	
(d)(i)	$\int \frac{A}{5-x} + \frac{B}{5+x} dx = p\ln(5-x) + q\ln(5+x)$	M1		integrate partial fractions, recognise logs their A and B; ignore the 1
	$= -A\ln(5-x) + B\ln(5+x)$	A1F		ft on A, B
				SC
				$\int \frac{1}{a^2-x^2} dx = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right)$ from FB
				$\int \frac{20}{5^2-x^2} dx = \frac{20}{10}\ln\left(\frac{5+x}{5-x}\right)$ M1A1
	$\int 1 dx = x$	A1		needs previous M mark
	$(1,3) \Rightarrow c = 2 + 2\ln 6 - 2\ln 4$	M1 A1	5	need to have c previously included accept $c = 2.8$ (2.81093..)
(ii)	when $x = 2$, $y = 3.116$	B1F	1	ft on sensible y (ln's and c) allow 3.11, 3.12
	Total		**14**	

MAP3 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$f(x) = (2-3x)^{-1}$	M1		
	$f'(x) = 3(2-3x)^{-2}$	M1A1		$-3(2-3x)^{-2}$ gets M1A0
	$f''(x) = 18(2-3x)^{-3}$	A1F	4	ft only on $f'(x) = -3(2-3x)^{-2}$
				SC Attempt to use quotient rule M1 $f'(x) = \frac{3}{(2-3x)^2}$ A1A1 $f''(x) = \frac{6x(\pm 3)(2-3x)}{(2-3x)^4}$ A1F ft only on earlier sign error
(ii)	$f(0) = \frac{1}{2}$ $f'(0) = \frac{3}{4}$ $f''(0) = \frac{18}{8}$	M1		use $x = 0$ in their derivatives
	$f(x) \approx \frac{1}{2} + \frac{3}{4}x + \frac{1}{2} \times \frac{9}{4}x^2$	A1	2	AG
(b)(i)	$\cos 2x = 1 - \frac{(2x)^2}{2}$	B1	1	or from first principles brackets possibly implied further down
(ii)	$\frac{1}{2} + \frac{3}{4}x + \frac{9}{8}x^2 = 1 - 2x^2 - 2x$	M1		Maclaurin series $=$ cos series $-2x$ condone missing $-2x$
		m1		attempt to manipulate line above to form $g(x) = 0$
	$25x^2 + 22x - 4 = 0$	A1		
	$x = 0.15(46\ldots)$	A1	4	ignore other answer
				SC if simplified quadratic omitted: $x = 0.15$ 2/4 $x = 0.154(6)$ 4/4
	Total		**11**	

MAP3 (cont)

Q	Solution	Marks	Total	Comments
6	$\int 400 - x^2 \ \mathrm{d}x = \int \frac{10}{\pi} t \ \mathrm{d}t$	M1		attempt to separate and integrate
	$400x - \frac{x^3}{3} = \frac{5t^2}{\pi} + c$	A1A1		A1 for each side; for both, need c somewhere or use of limits
	$t = 0, \ x = 6 \ \Rightarrow \ c = 2328$	M1A1F		ft on finding c, sensible error
	$x = 20 \quad t = 43.5$	M1A1	7	43, 44, 43,4, 43.456 **SC** use of $t = 0, \ x = 0$: allow M0 A0 M1 A0 max **SC** use of limits : $[\ldots]_6^{20} =$ M1 $\mathrm{f}(20) - \mathrm{f}(6) = 5t^2$ m1 $\pi(5333.33 - 2328) = 5t^2$ A1 $t = 43.5$ (or alternatives given above) A1
	Total		**7**	

MAP3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	M is (4, 1, 0)	B1		PI
	$\overrightarrow{BM}=\begin{bmatrix}2\\-3\\-1\end{bmatrix}$ $\overrightarrow{AC}=\begin{bmatrix}4\\0\\8\end{bmatrix}$	M1		$OC-OA$ or $OM-OB$ (or vice versa)
		A1F	3	ft on midpoint
				Alternative: $BM=BA+\frac{1}{2}AC$ (or $BC+\frac{1}{2}CA$) M1 $BA=OA-OB$ **or** $AC=OC-OA$ M1 $\overrightarrow{BM}=\begin{bmatrix}2\\-3\\-1\end{bmatrix}$ $\overrightarrow{AC}=\begin{bmatrix}4\\0\\8\end{bmatrix}$ CAO A1
(b)	$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\1\\-4\end{bmatrix}+\lambda\begin{bmatrix}4\\0\\8\end{bmatrix}+\mu\begin{bmatrix}2\\-3\\-1\end{bmatrix}$.	B1		r or $\begin{bmatrix}x\\y\\z\end{bmatrix}=OA$ or OB or OC or $OM\ldots$ $\ldots+\lambda AC+\mu BM$ OE
		B1F	2	ft on part (a)
(c)	$\begin{bmatrix}6\\5\\-3\end{bmatrix}\bullet\begin{bmatrix}4\\0\\8\end{bmatrix}=24-24=0$	M1A1		M1 for $\begin{bmatrix}6\\5\\-3\end{bmatrix}\bullet AC$ (OE from (b))
	$\begin{bmatrix}6\\5\\-3\end{bmatrix}\bullet\begin{bmatrix}2\\-3\\-1\end{bmatrix}=12-15+3=0$	A1E1	4	both scalar products zero, so perpendicular to plane
				Alternative 1: Rearrange (b) to $\mathbf{r}\bullet\mathbf{n}=d$ Attempt to eliminate λ,μ M1 λ,μ eliminated M1 $6x+5y-3z=29$ A1 normal is perpendicular to plane E1
				Alternative 2: cross product $\begin{bmatrix}4\\0\\8\end{bmatrix}\wedge\begin{bmatrix}2\\-3\\-1\end{bmatrix}=\begin{bmatrix}24\\20\\-12\end{bmatrix}=4\begin{bmatrix}6\\5\\-3\end{bmatrix}$ M1 attempt M1 A1 $a\wedge b$ is perpendicular to a, b E1
(d)	$\lvert BM\rvert=\sqrt{2^2+3^2+1^2}$	B1		
	$\lvert BD\rvert^2=t^2\left(6^2+5^2+(-3)^2\right)=\lvert BM\rvert^2$	M1		
	$t^2=\frac{14}{70}\left(=\frac{1}{5}\right)$	A1	3	AG
	Total		**12**	

	TOTAL		**60**	