GCE 2004
June Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics A
MAP3

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Key to Mark Scheme

M....mark is for .... method
m ....mark is dependent on one or more M marks and is for.... method
A....mark is dependent on M or m marks and is for .... accuracy
B....mark is independent of M or m marks and is for ....method and accuracy
E....mark is for ....explanation
√ or ft or F.... follow through from previous incorrect result
CAO....correct answer only
AWFW ....anything which falls within
AWRT ....anything which rounds to
AG .... answer given
SC .... special case
OE.... or equivalent
A2,1.... 2 or 1 (or 0) accuracy marks
-*x* EE....deduct *x* marks for each error
NMS....no method shown
PI ....possibly implied
SCA ....substantially correct approach
c.... candidate
SF....significant figure(s)
DP .... decimal place(s)

Abbreviations used in Marking

MC – *x*....deducted *x* marks for mis-copy
MR – *x*....deducted *x* marks for mis-read
ISW.... ignored subsequent working
BOD....given benefit of doubt
WR.... work replaced by candidate
FB .... formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working....mark as in scheme
Incorrect answer without working .... zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

MAP3

Q	Solution	Marks	Total	Comments
1(a)	$\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=\frac{-1}{2t^2}\frac{1}{2}$	M1		attempt $\frac{dy}{dt}$ & $\frac{dx}{dt}$; use $\frac{dy}{dt}\cdot\frac{dt}{dx}$ $\left(\frac{dy}{dt}\cdot\frac{dx}{dt}\text{ M0}\right)$
		A1	2	Use chain rule (ISW at this stage)
(b)	$t=1 \quad \frac{dy}{dx}=\frac{-1}{4}$	B1F		ft $t = 1$ subst in their $\frac{dy}{dx}$
	gradient of normal = 4	B1F		Follow on gradient $\frac{-1}{\text{their } -\frac{1}{4}}$
	$y=4x+c \quad t=1 \quad x=1 \quad y=\frac{1}{2}$	M1		Use $(1, \frac{1}{2})$ and gradient $\frac{1}{2}=\text{their }4+c$; $\frac{y-\frac{1}{2}}{x-1}=\text{their }4$
	$y=4x-\frac{7}{2}$	A1F	4	OE: F on gradient; y = (their m_N) $x + c$
	<u>Special Cases</u> **Eliminate *t* in part (a)** $y=\frac{1}{x+1}$; $\frac{dy}{dx}=\frac{\pm1}{(x+1)^2}$ M1 $=\frac{-1}{(2t)^2}$ A1 $m_T=-\frac{1}{4}$ B1F $\frac{1}{2}=-\frac{1}{4}\times1+c$; $c=\frac{3}{4}$ M1 $y=-\frac{1}{4}x+\frac{3}{4}$ A1F(5/6)			**Tangent instead of normal** $m_T=\frac{1}{4}$ $\frac{1}{2}=\frac{1}{4}\times1+c$; $c=\frac{1}{4}$ $y=\frac{1}{4}x+\frac{1}{4}$ (4/6)
	<u>Common error</u> $y=\frac{1}{2t}=2t^{-1}$; $\frac{dy}{dt}=2t^{-2}$; $\frac{dx}{dt}=2$ $\frac{dy}{dx}=\frac{2t^{-2}}{2}=-t^{-2}$ (or t^{-2}) M1A0 $m_N=-1, +1$ B1F $m_T = +1, -1$ B0F (no ft for just changing sign) $x=1, y=\frac{1}{2}$, $\frac{1}{2}=-1+c$ or $\frac{1}{2}=1+c$ M1			**NB** late substitution for t (could be retrospective) B1F B1F but if t's in final answer & no subst'n: either 0/4 or 1/4 if (1,1/2) and gradient used in linear equation

MAP3 (Cont)

Q	Solution	Marks	Total	Comments
1 cont	**Special Cases (cont)** **lns in** $\frac{dy}{dx}$ $x = 2t - 1 \quad y = \frac{1}{2t}$ $\frac{dx}{dt} = 2 \qquad \frac{dy}{dt} = \frac{\ln t}{2}$ $\frac{dy}{dx} = \frac{\ln t}{2} \cdot \frac{1}{2} = \frac{\ln t}{4}$ M1A0 $t = 1, \ \frac{dy}{dx} = 0$ B1F $m_T = 0, \quad m_N = \infty$ (normal is) $x = 1$ (3/4) $\begin{pmatrix}\text{tangent is}\\ \text{normal is}\end{pmatrix} y = \frac{1}{2}$ (2/4)			$\frac{dx}{dt} = \ln 2t$ $\frac{dy}{dx} = \frac{\ln 2t}{2}$ $\frac{dy}{dx} = \frac{\ln 2}{2}$ $m_T = \frac{\ln 2}{2}, \ m_N = \frac{-2}{\ln 2}$ B1F1F $\frac{1}{2} = \frac{-2}{\ln 2} + c$ M1 $y = \frac{-2}{\ln 2}x + \frac{1}{2} + \frac{2}{\ln 2}$ A1F
	Total		**6**	
2(a)	$(1+x)^{\frac{1}{3}} = 1 + \frac{1}{3}x + \frac{1}{3}\left(\frac{1}{3} - 1\right)\frac{x^2}{2}$	M1		
	$= 1 + \frac{1}{3}x - \frac{1}{9}x^2$	A1	2	
(b)	$(8+4x)^{\frac{1}{3}} = \left(8\left(1 + \frac{1}{2}x\right)\right)^{\frac{1}{3}}$	B1		
	$= 2\left(1 + \frac{1}{3}\frac{1}{2}x - \frac{1}{9}\left(\frac{1}{2}x\right)^2 + \ldots\right)$	M1		M1 for expression inside bracket SC: $(8 + 4x)^{\frac{1}{3}}$ $= 8^{\frac{1}{3}} + \frac{1}{3}8^{-\frac{2}{3}}.4x + \frac{1}{3}\left(-\frac{2}{3}\right)8^{-\frac{5}{3}}\frac{(4x)^2}{2}$ [M1 for $8^{\frac{1}{3}}$ $8^{-\frac{2}{3}}$, $8^{-\frac{5}{3}}$ M1 for $4x, \frac{(4x)^2}{2}$] $= 2 + \frac{1}{3}x - \frac{1}{18}x^2$
	$= 2 + \frac{1}{3}x - \frac{1}{18}x^2 + \ldots$	A1	3	Accept recurring decimals or equiv fractions
	Total		**5**	

MAP3 (Cont)

Q	Solution	Marks	Total	Comments
3(a)	$30 = A(7-2x)+B(x+4)$	M1		PFs: any valid method
	$x = -4 \quad 30 = 15A \qquad A = 2$	M1		for substituting values of x to find A, B
	$x = \frac{7}{2} \qquad 30 = \frac{15}{2}B \quad B = 4$	A1	3	
(b)	$\int_0^3 \frac{2}{x+4}+\frac{4}{7-2x}\,dx$			
	$= [2\ln(x+4)-2\ln(7-2x)]_0^3$	M1A1F		M1 for $[c\ln(x+4)+d\ln(7-2x)]$ Ignore limits here
	$= 2\ln 7 - 2\ln 1 - 2\ln 4 + 2\ln 7$	m1A1F		m1 for $(c\ln 7 + d\ln 1) - (c\ln 4 + d\ln 7)$ m1 Use limits right way round. A1 All correct and with ln1 = 0. A1F for $c\ln 7 - d\ln 7 - c\ln 4$
		A1	5	or $-2\ln\frac{4}{49}$ or $-4\ln\frac{2}{7}$ or $-1\ln\frac{16}{2401}$ or $1\ln\frac{2401}{16}$
	Total		**8**	

MAP3 (Cont)

Q	Solution	Marks	Total	Comments
4(a)	$9(y+2)^2 = 5+4(x-1)^2$			
	$x=2 \quad 9(y+2)^2 = 5+4$	M1		Substitute $x = 2$
				$9(y+2)^2 = 5 + 4 \times 3^2$ i.e. $(x+1)^2$
	$y+2=\pm1 \qquad y=-1,-3$	m1A1	3	Find two y values. Coords not required
				$(y+2)^2 = \frac{41}{9}, y+2 = \pm\frac{\sqrt{41}}{3}$ M1A0
(b)	$\frac{d}{dx}\left(9(y+2)^2\right) = \frac{d}{dx}\left(5 + 4(x-1)^2\right)$	M1		Attempt implicit differentiation with use of chain rule: $\frac{dy}{dx}$ attached to y term, not x term
	$18(y+2)\frac{dy}{dx} = 0 + 8(x-1)$	A1A1		
	$(2,-1) \quad (2,-3)$	m1		Use $x = 2$ and candidate's y values
	$\frac{dy}{dx} = \frac{4}{9} \qquad \frac{dy}{dx} = -\frac{4}{9}$	A1	5	OE; CAO
				<u>Alternative: explicit differentiation</u>
				$y = \sqrt{\frac{5+4(x-1)^2}{9}} - 2$
				$\frac{dy}{dx} = \frac{1}{2}\left(\frac{5+4(x-1)^2}{9}\right)^{-\frac{1}{2}} \frac{8}{9}(x-1)$
				(M1A2 fully correct; M1A1 if 9 of $\frac{8}{9}$ missing
				$x=2$: $\frac{dy}{dx} = \pm\frac{1}{2}(1)\frac{8}{9} = \pm\frac{4}{9}$
	Total		**8**	
5(a)	$V = \frac{1}{3}\pi r^2 h$ and $r = h$ (both)			
	$\Rightarrow V = \frac{1}{3}\pi h^3$	B1	1	AG
(b)	$\frac{dV}{dt} = 3$	B1		
	$3 = \pi h^2 \frac{dh}{dt}$	M1		Use $\frac{dV}{dh}$ in chain rule
	$h=2 \qquad \frac{dh}{dt} = 0.24$ (cm / min)	A1	3	CAO; Condone omission of units unless candidate converts to some other units.
	Total		**4**	

MAP3 (Cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\mathrm{f}(x)=\sin\left(2x+\frac{\pi}{6}\right)$			**Alternative** $\mathrm{f}(x)=\sin\left(2x+\frac{\pi}{6}\right)$
	$\mathrm{f}'(x)=2\cos\left(2x+\frac{\pi}{6}\right)$	M1A1		M1 for $\cos\left(2x+\frac{\pi}{6}\right)$; $=\sin 2x\cos\frac{\pi}{6}+\cos 2x\sin\frac{\pi}{6}$
	$\mathrm{f}''(x)=-4\sin\left(2x+\frac{\pi}{6}\right)$	A1	3	$=\frac{\sqrt{3}}{2}\sin 2x+\frac{1}{2}\cos 2x$ $\mathrm{f}'(x)=\sqrt{3}\cos 2x-\sin 2x$ M1A1 $\mathrm{f}''(x)=2\sqrt{3}\sin 2x-2\cos 2x$ A1 (cos $\frac{\pi}{6}$ & sin $\frac{\pi}{6}$ terms need not be simplified) If $\mathrm{f}(x)=\sin\left(2x+\frac{\pi}{6}\right)$ expanded incorrectly i.e. $=\sin 2x+\sin\frac{\pi}{6}$ $\mathrm{f}'(x)=2\cos 2x$ $\mathrm{f}''(x)=-4\sin 2x$, must be fully correct for M1A0A0 If $\mathrm{f}'(x)=\cos\left(2x+\frac{\pi}{6}\right)$ M1A0 $\mathrm{f}''(x)=-2\sin\left(2x+\frac{\pi}{6}\right)$ A1F $x=0$, $\mathrm{f}(0)=\frac{1}{2}$, $\mathrm{f}'(0)=\frac{\sqrt{3}}{2}$, $\mathrm{f}'(0)=-\frac{1}{2}$ M1A0 for part (ii)
(ii)	$\mathrm{f}(x)=\mathrm{f}(0)+\mathrm{f}'(0)x+\mathrm{f}''(0)\frac{x^2}{2}$			
	$\mathrm{f}(x)=\frac{1}{2}+2\frac{\sqrt{3}}{2}x-4\frac{1}{2}\frac{x^2}{2}$	M1		Use $x=0$ in Maclaurin series, P.I.
	$\mathrm{f}(x)\approx\frac{1}{2}+\sqrt{3}x-x^2$	A1	2	AG convincingly obtained: show how x^2 term is obtained
(b)	$\left(1-\left(1-\frac{x^2}{2}\right)\right)$	B1		Use of $\cos x=1-\frac{x^2}{2}$; may be derived from first principles
	$\left(\frac{1}{2}+\sqrt{3}x-x^2\right)\frac{x^2}{2}\approx\frac{1}{4}x^2$ $k=\frac{1}{4}$	M1A1	3	Either $k=\frac{1}{4}$ explicity stated or expression in question written with k replaced by $\frac{1}{4}$
	Total		**8**	

MAP3 (Cont)

Q	Solution	Marks	Total	Comments
7(a)	$\int \frac{dx}{x} = \int (1-kt)dt$	M1		Attempt to separate and integrate. M0 if mixture of x's and t's
	$\ln x = t - \frac{1}{2}kt^2 + c$	A1A1		c required
	$x = e^{t-\frac{1}{2}kt^2+c}$	M1		**Alternatives**
	$x = 2000,\ t = 0 \Rightarrow A = 2000$	M1		(1) $c = \ln 2000$ M1
	$x = Ae^{t-\frac{1}{2}kt^2}$, where $A = e^c$	A1	6	$\ln \frac{x}{2000} = t - \frac{1}{2}kt^2$
	(if A suddenly appears without justification: A0)			$\frac{x}{2000} = e^{t-\frac{1}{2}kt^2}$ M1
				$x = 2000\,e^{t-\frac{1}{2}kt^2}$ A1
				(2) $c = \ln 2000$ M1
				$x = e^{t-\frac{1}{2}kt^2} + \ln 2000$ M1
				$= e^{t-\frac{1}{2}kt^2}\ e^{\ln 2000}$
				$= 2000\ e^{t-\frac{1}{2}kt^2}$ A1
				(3) $\int_0^t (1-kt)dt$ M1
				$[\ln x]_{2000}^{x} = \left[t - \frac{1}{2}kt^2\right]_0^t$ — A1 for $\ln x$; A1 for $t - \frac{1}{2}kt^2$; A1 For both sets of limits
				$\ln x - \ln 2000 = t - \frac{1}{2}kt^2$ M1
				$\ln\left(\frac{x}{2000}\right) = t - \frac{1}{2}kt^2$ A1
				$x = 2000\,e^{t-\frac{1}{2}kt^2}$ AG AG convincingly obtained
(b)	Substituting $t = 12$ $x = 2000$	B1		No simplification required
	$12 - \frac{1}{2}k(12)^2 = \ln 1$	M1		For taking ln
	$k = \frac{1}{6}$	A1	3	OE
	Total		**9**	

MAP3 (Cont)

Q	Solution	Marks	Total	Comments
8(a)	$\overrightarrow{AB}=\begin{bmatrix}2\\0\\2\end{bmatrix}-\begin{bmatrix}3\\-1\\2\end{bmatrix}=\begin{bmatrix}-1\\1\\0\end{bmatrix}$	M1		
	l_1 has equation $\mathbf{r}=\begin{bmatrix}3\\-1\\2\end{bmatrix}+\lambda\begin{bmatrix}-1\\1\\0\end{bmatrix}$.	A1	2	OE eg $\mathbf{r}=\begin{bmatrix}2\\0\\2\end{bmatrix}+\lambda\begin{bmatrix}1\\-1\\0\end{bmatrix}$
(b)	$3-\lambda=4+\mu$ $-1+\lambda=1$ $2=-1-\mu$	M1		Set up at least 2 equations and attempt to solve.
	$\lambda=2 \quad \mu=-3$	A1		
	Confirm in third equation	A1		
	Intersect at (1, 1, 2)	A1	4	Alternative: showing (1, 1, 2) lies on both lines A2
(c)	$\begin{bmatrix}4\\1\\-1\end{bmatrix}+\mu\begin{bmatrix}1\\0\\-1\end{bmatrix}=\begin{bmatrix}9\\1\\-6\end{bmatrix}$.	M1		
	is satisfied by $\mu=5$	A1	2	
(d)	$\overrightarrow{CD}\bullet\overrightarrow{AB}=0$	B1		$\overrightarrow{CD}\cdot\begin{bmatrix}-1\\1\\0\end{bmatrix}=0$ or $\overrightarrow{CD}\cdot\begin{bmatrix}1\\-1\\0\end{bmatrix}=0$
	$\left(\begin{bmatrix}3\\-1\\2\end{bmatrix}+\lambda\begin{bmatrix}-1\\1\\0\end{bmatrix}-\begin{bmatrix}9\\1\\-6\end{bmatrix}\right)\bullet\begin{bmatrix}-1\\1\\0\end{bmatrix}=0$	M1		not $\overrightarrow{CD}\cdot l_1$, unless corrected later
	$(-6-\lambda)(-1)+(-2+\lambda)=0$	m1		
	$\lambda=-2 \quad D$ is $(5,-3,2)$	A1	4	Answer may be in vector form
				Alternative to part(d) $\begin{bmatrix}x-9\\y-1\\z+6\end{bmatrix}\cdot\begin{bmatrix}-1\\1\\0\end{bmatrix}=0$ B1 $\Rightarrow x-y=8$ M1 $\begin{bmatrix}x\\y\\z\end{bmatrix}=$ their $\mathbf{r}$ from (a) M1 (5, –3, 2) A1
	Total		**12**	
	Total		**60**	