GCE 2005

January Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics A

(MAP3)

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Key to Mark Scheme

M .... mark is for .... method
m .... mark is dependent on one or more M marks and is for .... method
A .... mark is dependent on M or m marks and is for .... accuracy
B .... mark is independent of M or m marks and is for .... method and accuracy
E .... mark is for .... explanation
√ or ft or F .... follow through from previous incorrect result
CAO .... correct answer only
AWFW .... anything which falls within
AWRT .... anything which rounds to
AG .... answer given
SC .... special case
OE .... or equivalent
A2,1 .... 2 or 1 (or 0) accuracy marks
-*x* EE .... deduct *x* marks for each error
NMS .... no method shown
PI .... possibly implied
SCA .... substantially correct approach
c .... candidate
SF .... significant figure(s)
DP .... decimal place(s)

Abbreviations used in Marking

MC – *x* .... deducted *x* marks for mis-copy
MR – *x* .... deducted *x* marks for mis-read
ISW .... ignored subsequent working
BOD .... given benefit of doubt
WR .... work replaced by candidate
FB .... formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working .... mark as in scheme
Incorrect answer without working .... zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

MAP3

Q	Solution	Marks	Total	Comments
1(a)	$x=\frac{\sqrt{3}}{2},\ y=1$ both	B1	1	Accept $x = 0.866$
(b)(i)	$\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=\frac{-2\sin t}{\cos t}$	M1A1	2	
(ii)	Grad at $P=-2\sqrt{3}$	B1F	1	Accept – 3.46, $-\sqrt{12}$; ft $\frac{dy}{dx}$ and consistent errors in $\sin\frac{\pi}{3}$ and/or $\cos\frac{\pi}{3}$
(c)	$y-1=-2\sqrt{3}\left(x-\frac{\sqrt{3}}{2}\right)$	M1		OE SC A1F on grad = $a\sqrt{3}$ max. 5/6
	$y=-2\sqrt{3x}+4$	A1	2	Accept $y = -3.46x + 4$ AWRT
	Total		**6**	

MAP3 (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)	$(1+x)^{-1}$			
	$=1+-1x+\frac{-1.-2}{2!}x^2+\frac{-1.-2.-3}{3!}x^3.$	M1		
	$=1-x+x^2-x^3\ldots$	A1	2	
(ii)	$\frac{1}{(3+2x)}=\frac{1}{3}(\ldots)$	B1		Alternative – use of $(a+x)^n$
				$(3+2x)^{-1}=3^{-1}+-1\times3^{-2}(2x)+$
	$x\to\frac{2}{3}x\Rightarrow 1-\frac{2}{3}x+\frac{4}{9}x^2-\frac{8}{27}x^3$	M1		$\frac{-1-2-3^{-3}(2x)^2}{2!}+\frac{-1-2-3-3^{-4}(2x)^3}{3!}$
				powers of 3, and $(2x)$ M1
				$n=-1$, and factorials M1
	$\left(1+\frac{2}{3}x\right)^{-1}=$			all correct A1
	$\frac{1}{3}-\frac{2}{9}x+\frac{4}{27}x^2-\frac{8}{81}x^3$	A1	3	AG convincing by obtained
2(b)	$8+7x=A(3+2x)+B(1+x)$	M1		
	$x=-1$ $\quad$ $x=-\frac{3}{2}$	M1		
	$A=1$ $\quad$ $B=5$	A1	3	
(c)	$(1-x+x^2-x^3)+$	M1		
	$5\left(\frac{1}{3}-\frac{2}{9}x+\frac{4}{27}x^2-\frac{8}{81}x^3\right)$	A1F		ft A and B and expansions
	$=\left(\frac{8}{3}-\frac{19}{9}x+\frac{47}{27}x^2-\frac{121}{81}x^3\right)$	A1	3	OE; CAO
	Total		**11**	

MAP3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$t = 7$	M1		
	$P = 90\times1.12^7 = 198.9\ldots = 199$	A1	2	AG
(b)	$k^7 = 1.5$	M1		
	$k = \sqrt[7]{1.5}$ or $7\ln k = \ln 1.5$	m1		
	$k = 1.059\ldots$	A1	3	AG
(c)	$P = Q \Rightarrow \frac{1}{3} = \frac{1.06^t}{1.12^t}$	M1		Or reciprocal.
				t on one side of correct equation with $\frac{270}{90} = 3$.
	$t\ln\left(\frac{1.12}{1.06}\right) = \ln 3$	m1		OE
	$t = 19.95$	A1		Accept range 19.83 to 19.95
	$1998 + 19 = 2017$	B1F	4	ft t condone 2018 SC trial and improvement Accept $\frac{2017}{18}$ for B1 only
	Total		**9**	

MAP3 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$f(x)=e^{-3x}$ $f'(x)=-3e^{-3x}$ $f''(x)=9e^{-3x}$	M1A1	2	
(ii)	$f(x)=f(0)+f'(0)x+f''(0)\dfrac{x^2}{2!}\ldots$	M1		
	$f(0)=1$ $f'(0)=-3$ $f''(0)=9$			
	$f(x)\approx 1-3x+\dfrac{9}{2}x^2$	A1	2	AG. Use of Maclaurin from (i) required.
(b)	$\ln(1+3x)\approx 3x-\dfrac{(3x)^2}{2}+\dfrac{(3x)^3}{3}$	M1		Allow $3x-\dfrac{3x^2}{2}+\dfrac{3x^3}{3}\left(\text{or } x^3\right)$
	$=3x-\dfrac{9}{2}x^2+9x^3$	A1	2	CAO but allow $\dfrac{27}{3}x^3$
(c)	$3x-\dfrac{9}{2}x^2+9x^3-\left(2x-6x^2+9x^3\right)=0.1$	M1		
	$1.5x^2+x-0.1=0$	A1F		ft $\ln(1+3x)$ and simplification to $f(x)=0$. Correct quadratic any equivalent form
	$x=\dfrac{-1+\sqrt{1.6}}{3}=0.088$	M1A1	4	
	Total		**10**	

MAP3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	40 cm sec^{-1} or $\frac{\text{d}r}{\text{d}t} = 40$	B1		
	$t = 2 \quad r = 40t + 50 = 130$	B1	2	
(b)(i)	$\frac{\text{d}r}{\text{d}t} = \frac{k}{r} \qquad \int r\text{d}r = \int k\text{d}t$	M1		Using limits $\int r\text{d}r = \int k\text{d}t$
	$\frac{1}{2}r^2 = kt + c$	A1		$\left[\frac{1}{2}r^2\right]_{50}^{250} = [kt]_0^5$
	$t = 0; \quad r = 50 \qquad \frac{1}{2}r^2 = kt + 1250$	M1		$\frac{1}{2}\left[250^2 - 50^2\right] = 5k$
	$t = 5; \quad r = 250 \qquad 5k = 31250 - 1250$			
	$k = 6000$	A1	4	AG $\qquad k = 6000$
(ii)	$r^2 = 26500$ $r = 162.8 \approx 163$	B1F	1	ft sensible equation for r. (c found in (i)) AWRT
(iii)	$\frac{\text{d}A}{\text{d}t} = \frac{\text{d}A}{\text{d}r}\frac{\text{d}r}{\text{d}t}$ or $A = \pi(2kt + 2500)$	M1		Chain rule in A, r, t. OE
	$\frac{\text{d}A}{\text{d}t} = 2\pi r \times \frac{k}{r} \qquad \frac{\text{d}A}{\text{d}t} = \pi \times 2k$			
	$= 2\pi k$	A1		12000π
	which is constant as k is constant	E1	3	which is constant
	Total		**10**	

MAP3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\overrightarrow{AB}=\begin{bmatrix}2\\-3\\-2\end{bmatrix}$	M1		
	$r=\begin{bmatrix}3\\5\\1\end{bmatrix}+\lambda\begin{pmatrix}2\\-3\\-2\end{pmatrix}$	A1	2	$r=$ or $\begin{pmatrix}x\\y\\z\end{pmatrix}=$ required.
(b)	$2x+y-3z=1$			
	At C, $(2\times1)+8-(3\times3)=2+8-9=1$	B1		or $\begin{pmatrix}2\\1\\-3\end{pmatrix}.\begin{pmatrix}1\\8\\3\end{pmatrix}=2+8-9=1$
	$3+2\lambda=1$ $\lambda=-$ $5-3\lambda=8$ $\lambda=-$ $1-2\lambda=3$ $\lambda=-$ $\begin{pmatrix}3\\5\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\-3\\-2\end{pmatrix}=\begin{pmatrix}1\\8\\3\end{pmatrix}$	B1 E1	3	$\lambda=-1$ $\lambda=-1$ stated as verifying vector equation or the 3 component equations seen.
(c)(i)	Line AD is $r=\begin{bmatrix}3\\5\\1\end{bmatrix}+t\begin{pmatrix}2\\1\\-3\end{pmatrix}$	B1 B1	2	$r=\begin{bmatrix}3\\5\\1\end{bmatrix}+tAD$ with AD in sensible col. form. $AD=\begin{bmatrix}2\\1\\-3\end{bmatrix}$
(ii)	At D, $2(3+2t)+(5+t)-3(1-3t)=1$	M1		
	$8+14t=1$ $t=-\frac{1}{2}$	A1		
	D is $\left(2,\frac{9}{2},\frac{5}{2}\right)$	A1	3	
(iii)	$\overrightarrow{AC}\cdot2\left(\overrightarrow{AD}\right)=\begin{bmatrix}-2\\3\\2\end{bmatrix}\cdot\begin{bmatrix}-2\\-1\\3\end{bmatrix}$	M1		$\pm$ correct vectors, or multiples.
	$=4-3+6=7$	A1		
	$\sqrt{17}\sqrt{14}\cos\theta=7$	M1		Correct scalar product formula between two vectors.
	$\cos\theta=0.4537...$ $\theta=63.0^\circ$	A1F	4	F on θ acute.
	Total		**14**	
	Total		**60**	