AQA

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark scheme January 2004

GCE

Mathematics A

Unit MAP3

Key to mark scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m mark and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
– x EE		Deduct x marks for each error
NMS		No method shown
PI		Perhaps implied
c		Candidate

Abbreviations used in marking

MC – x	deducted x marks for miscopy
MR – x	deducted x marks for misread
ISW	ignored subsequent working
BOD	gave benefit of doubt
WR	work replaced by candidate

Application of mark scheme

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Q	Solution	Marks	Total	Comments
1(a)(i)	$\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=6.\frac{1}{6t}$	M1 A1	 2	
(ii)	$t=\frac{1}{2}$ gradient $=2$	B1√	1	ft only on $\frac{dy}{dx}=f(t)$
(b)(i)	$t=\frac{y}{6}$ $x=3\left(\frac{y}{6}\right)^2=\left[\frac{y^2}{12}\right]$	M1A1	2	Accept $\frac{3y^2}{36}$ Use of tangent $y=2x+\frac{3}{2}$ or $x=\frac{y}{2}-\frac{3}{4}$ instead of curve: no marks
(ii)	$\frac{dx}{dy}=\frac{2y}{12}$ $t=\frac{1}{2}$ $y=3$ $\frac{dx}{dy}=\frac{6}{12}$ $\frac{dy}{dx}=\frac{12}{6}=2$	M1 B1 M1A1	 4	Alternative: $\frac{dx}{dy}=\frac{2y}{12}$ M1 $\frac{y}{6}=t$ B1 $t=\frac{1}{2};\frac{dx}{dy}=\frac{1}{2}$ M1 $\frac{dy}{dx}=2$ A1
	Total		**9**	
2	x y step x $\frac{dy}{dx}$ step y 1 0.5 0.25 0.5 0.125 1.25 0.625 0.25 0.3386 0.0846 1.5 0.7096 $x=0.71$	 M1A1 M1A1 A1	 5	$\frac{dy}{dx}=0.5$ M1 Step dy = 0.125 A1 1.25; step y + 0.5; step $y=0.25\frac{dy}{dx}$ M1 0.08 (46) AWRT A1
	Total		**5**	
3 (a)(i)	$t=0$ $P=50$	B1	1	
(ii)	$e^{\frac{-t}{4}}\to 0$ $P\to 100$	B1	1	
(b)	$75=100-50e^{\frac{-t}{4}}$ $\frac{1}{2}=e^{\frac{-t}{4}}$ $\ln\frac{1}{2}=\frac{-t}{4}$ $t=2.8$	M1A1 M1A1	 4	Allow $\frac{25}{50}$ for $\frac{1}{2}$ SC trial and improvement 2.8 4/4, 2.77 3/4 else 0
	Total		**6**	

Q		Solution	Marks	Total	Comments
4	(a)	$8 + 3x = A(2 - x) + B(1 + 3x)$	M1		Any equivalent method
		$x = 2 \quad 14 = 7B \quad B = 2$	M1		
		$x = \frac{-1}{3} \quad 7 = \frac{7}{3}A \quad A = 3$	A1	3	
	(b)	$\frac{1}{1+3x} = (1+3x)^{-1}$			Alternative by Maclaurin $f' = \frac{\pm 3}{(1+3x)^2}$; $f'' = \frac{\pm 18 \text{ or } 6}{(1+3x)^3}$ M1 and $f(0)$ $f'(0)$ $f''(0)$ seen
		$=1 + -1(3x) + \frac{-1.-2}{2}(3x)^2$	M1		Allow $3x^2$
		$=1 - 3x + 9x^2$	A1	2	
	(c)	$\frac{1}{2-x} = \frac{1}{2(1-\frac{x}{2})}$	B1		Alternative: $(2-x)^{-1} = 2^{-1} + (-1)2^{-2}(-x)$ $+\frac{(-1.-2)}{2!}2^{-3}(-x)^2$ M1 – use negative powers of 2 A1 – coefficients correct A1 – all correct, with use of $-x$ seen
		$= \left(1 + -1\left(\frac{-x}{2}\right) + \frac{-1.-2}{2}\left(\frac{-x^2}{2}\right)\right)$	M1		
		$= \frac{1}{2} + \frac{x}{4} + \frac{x^2}{8}$	A1	3	Answer given, convincingly obtained Alternative: $(1-\frac{x}{2})^{-1}$ by Maclaurin $f' = \frac{\pm 1}{(2-x)^2}$ $f'' = \frac{\pm 2}{(2-x)^3}$ M1 $f(0)$ $f'(0)$ $f''(0)$ seen M1 AG convincingly obtained A1
	(d)	$\frac{8+3x}{(1+3x)(2-x)}$			
		$= 3(1 - 3x + 9x^2) + 2\left(\frac{1}{2} + \frac{x}{4} + \frac{x^2}{8}\right)$	M1M1		M1 – use series M1 – use PFs and multiply out
		$4 - \frac{17}{2}x + \frac{109}{4}x^2$	A1	3	Alternative: $(8+3x)(1-3x+9x^2)\left(\frac{1}{2}+\frac{x}{4}+\frac{x^2}{8}\right)$ M1 Multiply out M1
	(e)	Valid for $\lvert x \rvert < \frac{1}{3}$	B2	2	B1 for $x < \frac{1}{3}$ B1 for $\lvert x \rvert < \frac{1}{3}$ and $\lvert x \rvert < 2$ or $\lvert x \rvert < 1$
		Total		**13**	

Q	Solution	Marks	Total	Comments
5 (a)(i)	$\text{f}(x) = \text{e}^{-2x}$ $\quad \text{f}'(x) = -2\text{e}^{-2x}$	B1		
	$\text{f}''(x) = 4\text{e}^{-2x}$	B1√	2	
(ii)	$\text{f}(x) = \text{f}(0) + \text{f}'(0)x + \text{f}''(0)\dfrac{x^2}{2}$			
	$\text{f}(0) = 1 \quad \text{f}'(0) = -2$	M1		Use $x = 0$ in Maclaurin series
	$\text{f}''(0) = 4$			
	$\text{f}(x) \approx 1 - 2x + 2x^2$	A1	2	AG convincingly obtained
(b)(i)	$\cos 3x \approx 1 - \dfrac{(3x)^2}{2}$	B1	1	
(ii)	$1 - 2x + 2x^2 = 1 - \dfrac{9}{2}x^2$	M1		Set up equation
	$\dfrac{13}{2}x^2 - 2x = 0$	m1		Rearrange to soluble form
	$x = \dfrac{4}{13}, \quad 0.308$	A1	3	Accept 0.31 Ignore $x = 0$
	Total		**8**	
6 (a)	$\displaystyle\int \frac{\text{d}v}{10 - 5v} = \int \text{d}t$	M1		Attempt to separate and integrate
	$-\dfrac{1}{5}\ln(10 - 5v) = t + c$	M1 A1A1		$\pm k\ln(10 - 5v)$ c required
	$t = 0 \quad v = 0 \quad c = -\dfrac{1}{5}\ln 10$	B1√		Find c or use limits
	$t = \dfrac{1}{5}\ln\left(\dfrac{10}{10 - 5v}\right) = \dfrac{1}{5}\ln\left(\dfrac{2}{2 - v}\right)$	A1	6	AG convincingly obtained
(b)	$\text{e}^{5t} = \dfrac{2}{2 - v}$	M1		Alternative: $0.5 = \dfrac{1}{5}(\ln 2 - \ln(2 - v))$ M1
	$t = 0.5 \quad 2 - v = 2\text{e}^{-2.5}$	m1		$\text{e}^{\ln 2 - 2.5} = \text{e}^{\ln(2 - v)}$ M1
	$v = 1.8358 \quad v = 1.8 \text{ m s}^{-1}$	A1	3	$v = 1.8$ A1
	Total		**9**	

Q	Solution	Marks	Total	Comments
7 (a)(i)	$\overrightarrow{AB}=\begin{bmatrix}2\\4\\-4\end{bmatrix}$ $\lvert\overrightarrow{AB}\rvert=\sqrt{2^2+4^2+4^2}=6$	M1A1	2	No marks for $\overrightarrow{AB}$ alone
(ii)	M is (4, 1, 0)	B1	1	Accept $\begin{bmatrix}4\\1\\0\end{bmatrix}$
(b)	$\overrightarrow{CM}\bullet\overrightarrow{AB}=\begin{bmatrix}-4\\3\\1\end{bmatrix}\bullet\begin{bmatrix}2\\4\\-4\end{bmatrix}$ $=-8+12-4=0$	M1A1	2	M1 – sensible attempt at $\overrightarrow{CM}\bullet\overrightarrow{AB}$ Allow $\overrightarrow{MC}$ for $\overrightarrow{CM}$ $\mp 8\mp 12\mp 4=0$ must be seen
(c)	$\mathbf{r}\bullet\begin{bmatrix}-4\\3\\1\end{bmatrix}=\begin{bmatrix}-4\\3\\1\end{bmatrix}\bullet\begin{bmatrix}3\\-1\\2\end{bmatrix}=-13$ $4x-3y-z=13$	M1A1	2	$\begin{bmatrix}3\\-1\\2\end{bmatrix}$ or $\begin{bmatrix}5\\3\\-2\end{bmatrix}$ or $\begin{bmatrix}4\\1\\0\end{bmatrix}$ **a** **b** **m** AG convincingly obtained
(d)	$4(8+5t)-3(-2-3t)-(-1+3t)=13$ $39+26t=13 \quad t=-1$ P is $(3, 1, -4)$	M1 m1 A1	3	Solve for t Accept $\begin{bmatrix}3\\1\\-4\end{bmatrix}$
	Total		**10**	
	Total		**60**	