

General Certificate of Education  
June 2005  
Advanced Level Examination



**MATHEMATICS (SPECIFICATION A)**  
**Unit Pure 2**

**MAP2**

Thursday 16 June 2005 Afternoon Session

**In addition to this paper you will require:**

- an 8-page answer book;
  - the AQA booklet of formulae and statistical tables.
- You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

**Information**

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

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1 The geometric sequence  $u_1, u_2, u_3, \dots$  is defined by

$$u_{n+1} = u_n + \frac{2}{3}3^n, \quad \text{where } u_1 = 1.$$

(a) Write down the values of  $u_2, u_3, u_4$  and  $u_5$ . (3 marks)

(b) Hence write down the  $n$ th term of this sequence in terms of  $n$ . (1 mark)

(c) Show that the sum of the first 100 terms of the sequence is approximately  $2.58 \times 10^{47}$ . (2 marks)

2 (a) Express  $\frac{4x+1}{2x-1}$  in the form  $A + \frac{B}{2x-1}$ , where  $A$  and  $B$  are constants. (3 marks)

(b) Write down the equations of the asymptotes to the graph of  $y = \frac{4x+1}{2x-1}$ . (2 marks)

(c) Sketch the graph of  $y = \frac{4x+1}{2x-1}$ , indicating clearly each asymptote and also the points of intersection of the graph with the coordinate axes. (3 marks)

(d) Hence, or otherwise, solve the inequality  $\frac{4x+1}{2x-1} < -1$ . (2 marks)

3 (a) Show that the equation of the normal to the curve  $y = x^3 - 7x + 6$  at the point  $P(1, 0)$  is  $4y - x + 1 = 0$ . (4 marks)

(b) The point  $R(2k, -k)$  lies on this normal.

Determine the value of  $k$ . (2 marks)

(c) Given that  $Q$  is the point  $(-3, 0)$ , find the area of the triangle  $PQR$ . (2 marks)

4 (a) Solve the equation  $\sec x = 2$  for  $0 \leq x \leq 2\pi$ . (2 marks)

(b) Use the identity  $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$  to show that

$$\cos 2x = 2 \cos^2 x - 1. \quad (2 \text{ marks})$$

(c) Hence solve the equation

$$\cos 2x + 3 \cos x - 1 = 0 \quad \text{for } 0 \leq x \leq 2\pi. \quad (5 \text{ marks})$$

5 (a) Find  $\frac{dy}{dx}$  when  $y = \frac{e^x}{\sin x}$ . (3 marks)

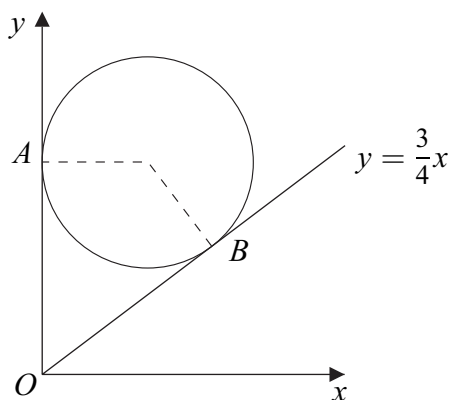
(b) (i) Verify that the graph of  $y = \frac{e^x}{\sin x}$  has a stationary point when  $x = \frac{\pi}{4}$ . (2 marks)

(ii) Find the value of  $y$  at this stationary point, giving your answer to three significant figures. (2 marks)

6 The circle  $C$  has the equation  $(x - 10)^2 + (y - 20)^2 = 100$ .

(a) State the radius, and the coordinates of the centre, of the circle  $C$ . (2 marks)

(b) The diagram shows the circle  $C$  and the line  $y = \frac{3}{4}x$  which is the tangent to this circle at the point  $B$ . The circle  $C$  also touches the  $y$ -axis at the point  $A$ .



(i) Write down the length of  $OA$  and of  $OB$ . (1 mark)

(ii) Show that the  $x$ -coordinate of  $B$  is 16. (2 marks)

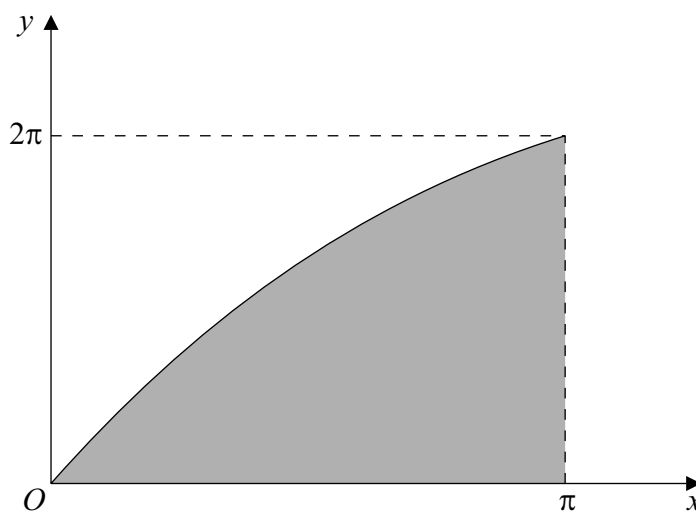
(iii) Calculate the length of the chord  $AB$ , writing your answer in the form  $p\sqrt{5}$ , where  $p$  is an integer. (3 marks)

7 (a) Use integration by parts to show that  $\int_0^{\pi} x \sin x \, dx = \pi$ . (4 marks)

(b) (i) Express  $\sin^2 x$  in terms of  $\cos 2x$ . (1 mark)

(ii) Hence show that  $\int_0^{\pi} \sin^2 x \, dx = \frac{\pi}{2}$ . (3 marks)

(c) The diagram below shows the graph of  $y = 2x + \sin x$  for  $0 \leq x \leq \pi$ .



Calculate the volume of revolution of the solid formed when the shaded region is rotated through  $2\pi$  radians about the  $x$ -axis.

Give your answer to three significant figures. (4 marks)

**END OF QUESTIONS**