General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) MAP2
Unit Pure 2

Wednesday 16 June 2004 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- an insert for use in Question 5 (enclosed);
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used, including the insert for use in Question 5, to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Find the coordinates of the points where the graph of $y = \dfrac{2x+1}{x-1}$ crosses:

(i) the y-axis; *(1 mark)*

(ii) the x-axis. *(1 mark)*

(b) (i) Express $\dfrac{2x+1}{x-1}$ in the form $A + \dfrac{B}{x-1}$. *(3 marks)*

(ii) Write down the equations of the two asymptotes to the graph of $y = \dfrac{2x+1}{x-1}$. *(2 marks)*

(c) Hence sketch the graph of $y = \dfrac{2x+1}{x-1}$. *(3 marks)*

(d) By using your graph, or otherwise, solve the inequality

$$\frac{2x+1}{x-1} \leqslant 0\,. \qquad \textit{(2 marks)}$$

2 (a) Show that $\sin(\alpha+\beta) + \sin(\alpha-\beta) \equiv 2\sin\alpha\cos\beta$. *(2 marks)*

(b) (i) Express $2\sin 8x\cos 2x$ in the form $\sin A + \sin B$. *(2 marks)*

(ii) Hence find $\displaystyle\int 6\sin 8x\cos 2x\,\mathrm{d}x$. *(3 marks)*

3 (a) Use integration by parts to evaluate $\displaystyle\int_0^{\frac{\pi}{2}} x\cos x\,\mathrm{d}x$. *(5 marks)*

(b) (i) Use the substitution $t = x^2 + 4$ to show that $\displaystyle\int \frac{2x\,\mathrm{d}x}{\sqrt{x^2+4}} = \int \frac{1}{\sqrt{t}}\,\mathrm{d}t$. *(2 marks)*

(ii) Show that $\displaystyle\int_0^2 \frac{2x\,\mathrm{d}x}{\sqrt{x^2+4}} = 4(\sqrt{2}-1)$. *(4 marks)*

4 (a) (i) Find $\frac{dy}{dx}$ when $y = e^x \sin 2x$. *(3 marks)*

(ii) Hence find the equation of the tangent to the curve $y = e^x \sin 2x$ at the origin. *(2 marks)*

(b) Show that the equation of the normal to the curve $y = e^x \sin 2x$ at the point where $x = \pi$ is

$$2e^{\pi}y + x = \pi. \quad \textit{(4 marks)}$$

5 [**Figure 1**, printed on the insert, is provided for use in answering this question.]

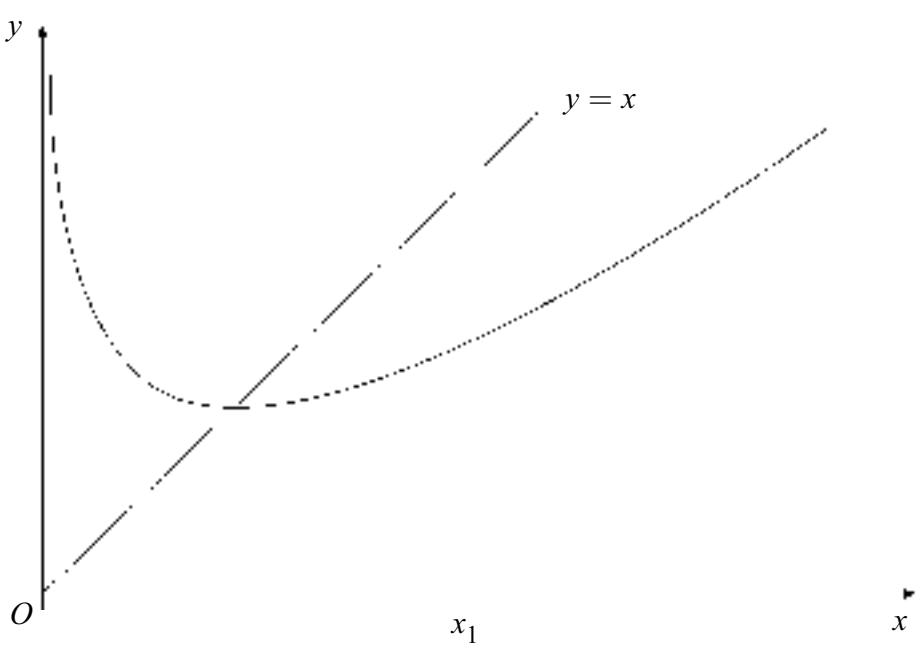
(a) Show, without using a calculator, that the equation $x^3 - 15 = 0$ has a root in the interval $2 \leqslant x \leqslant 3$. *(2 marks)*

(b) (i) Show that the equation $x = \frac{2x}{3} + \frac{5}{x^2}$ can be rearranged to give the equation

$$x^3 - 15 = 0. \quad \textit{(2 marks)}$$

(ii) Use the iterative formula $x_{n+1} = \frac{2x_n}{3} + \frac{5}{x_n^2}$, starting with $x_1 = 3$, to find the values of x_2, x_3 and x_4, giving your answers to six decimal places. *(4 marks)*

(iii) The graphs of $y = \frac{2x}{3} + \frac{5}{x^2}$ and $y = x$ are sketched below.



On **Figure 1**, draw a staircase diagram to illustrate the convergence of the sequence $x_1, x_2, x_3, \ldots$ *(2 marks)*

(iv) Write down the **exact** value to which this sequence converges. *(1 mark)*

6 (a) The circle $(x-4)^2+(y-3)^2=4$ has centre C and radius r.

Write down:

(i) the coordinates of C;

(ii) the value of r. *(2 marks)*

(b) The line $y=x+1$ intersects this circle at two points A and B.

(i) Find the coordinates of A and B. *(5 marks)*

(ii) Show that the area of the minor segment bounded by the circle and the chord AB is $\pi-2$. *(3 marks)*

END OF QUESTIONS

Surname		Other Names	
Centre Number						Candidate Number				
Candidate Signature										

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Insert for use in answering Question 5.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

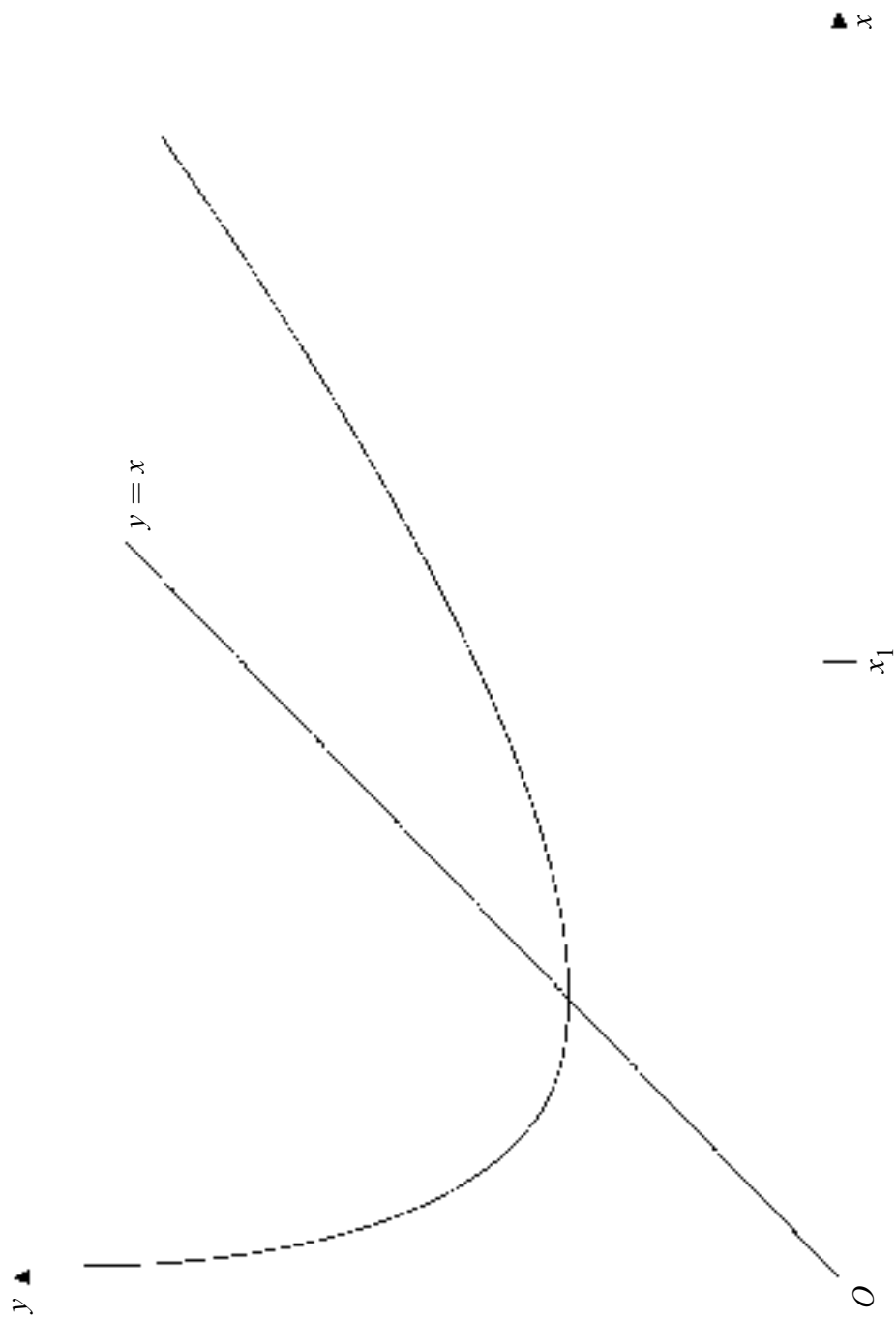

Figure 1 (for Question 5)