

General Certificate of Education  
January 2005  
Advanced Level Examination



**MATHEMATICS (SPECIFICATION A)**  
**Unit Pure 2**

**MAP2**

Friday 21 January 2005 Afternoon Session

**In addition to this paper you will require:**

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

**Information**

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

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1 The quadratic equation  $x^2 - 3x + 9 = 0$  has roots  $\alpha$  and  $\beta$ .

(a) Write down the numerical values of:

(i)  $\alpha + \beta$ ; (1 mark)

(ii)  $\alpha\beta$ . (1 mark)

(b) Hence find the numerical values of:

(i)  $\frac{6}{\alpha} \times \frac{6}{\beta}$ ; (1 mark)

(ii)  $\frac{6}{\alpha} + \frac{6}{\beta}$ . (2 marks)

(c) Hence, or otherwise, find the quadratic equation with roots  $\frac{6}{\alpha}$  and  $\frac{6}{\beta}$ , in the form  $x^2 + bx + c = 0$ , where  $b$  and  $c$  are integers. (2 marks)

2 (a) Show that the equation  $xe^x - 5 = 0$  has a root in the interval  $1 < x < 2$ . (2 marks)

(b) Differentiate  $xe^x$  with respect to  $x$ . (2 marks)

(c) Using the Newton–Raphson method **once**, with an initial value for  $x$  of 1.2, find an approximation to the root of the equation  $xe^x - 5 = 0$  in the interval  $1 < x < 2$ , giving your answer to three decimal places. (3 marks)

3 The function  $f$  is given by

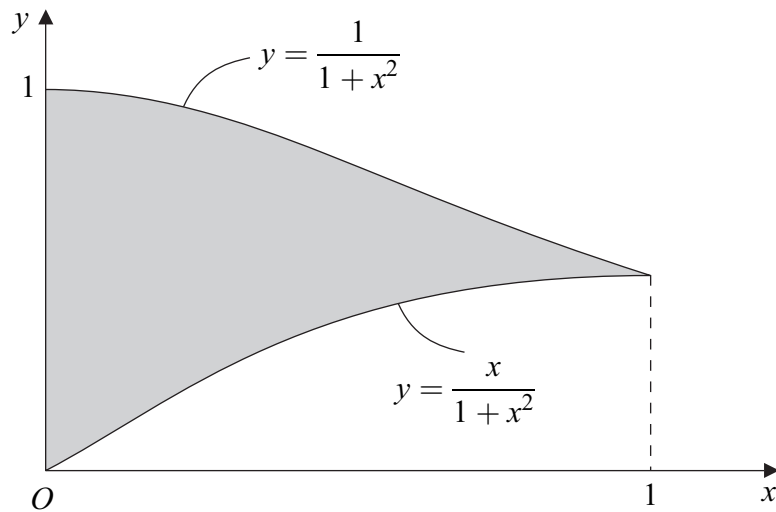
$$f(x) = x^3 + ax^2 + bx + 6.$$

When  $f(x)$  is divided by  $(x - 1)$ , the remainder is 24.

When  $f(x)$  is divided by  $(x + 2)$ , the remainder is also 24.

Use the Remainder Theorem to find the numerical values of  $a$  and  $b$ . (4 marks)

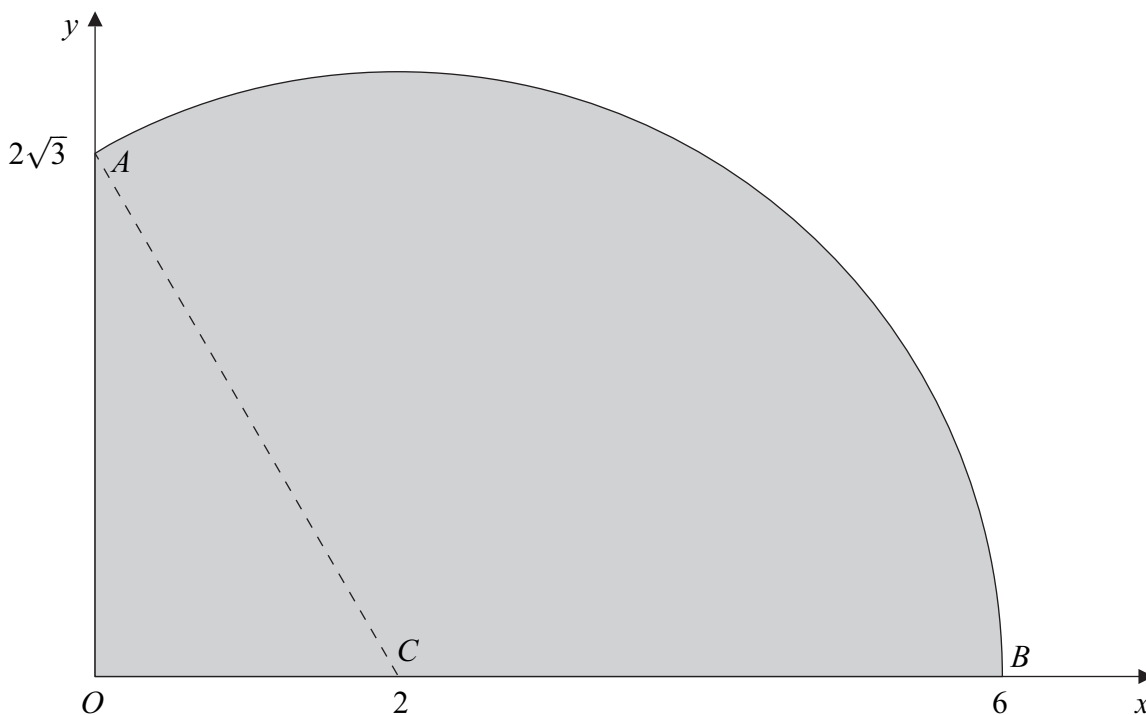
- 4 (a) (i) Differentiate  $\ln(1 + x^2)$  with respect to  $x$ . (2 marks)
- (ii) Hence, or otherwise, evaluate  $\int_0^1 \frac{x}{1 + x^2} dx$ . (2 marks)
- (b) (i) Given that  $y = \tan^{-1} x$ , express  $x$  in terms of  $y$ . (1 mark)
- (ii) Hence find  $\frac{dx}{dy}$  in terms of  $y$ . (1 mark)
- (iii) Hence, using an appropriate trigonometrical identity, show that  $\frac{dy}{dx} = \frac{1}{1 + x^2}$ . (2 marks)
- (iv) Evaluate  $\int_0^1 \frac{1}{1 + x^2} dx$ . (2 marks)
- (c) The diagram shows the graphs of  $y = \frac{1}{1 + x^2}$  and  $y = \frac{x}{1 + x^2}$  for  $0 \leq x \leq 1$ .  
The graphs intersect at the point where  $x = 1$ .



Show that the area of the shaded region is  $\frac{\pi}{4} - \frac{1}{2} \ln 2$ . (2 marks)

5 The diagram shows the graph of  $y = \sqrt{16 - (x - 2)^2}$  for  $x \geq 0$  and  $y \geq 0$ .

The points of intersection of the curve with the coordinate axes are  $A(0, 2\sqrt{3})$  and  $B(6, 0)$ .



- (a) Use the trapezium rule, with six strips, to estimate the area of the shaded region. Give your answer to one decimal place. (4 marks)
- (b) The curve forms part of a circle with centre  $C(2,0)$ .
- (i) Write down the radius of the circle. (1 mark)
- (ii) Show that the angle  $ACB$  is  $120^\circ$ . (2 marks)
- (c) (i) Find the area of the sector  $ACB$ . (1 mark)
- (ii) Hence show that the area of the shaded region is  $\frac{16\pi}{3} + 2\sqrt{3}$ . (2 marks)
- (d) The shaded region is rotated about the  $x$ -axis through  $360^\circ$  to form a solid of revolution. Use integration to find the volume of this solid. (4 marks)

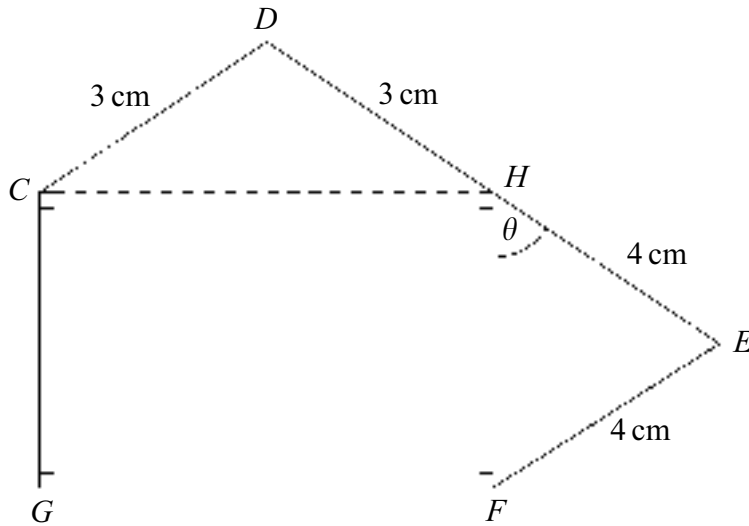
- 6 (a) Express

$$6 \sin \theta + 8 \cos \theta$$

in the form  $R \sin(\theta + \alpha)$ , where  $R > 0$  and  $\alpha < \frac{\pi}{2}$ , giving the value of  $\alpha$  to three decimal places. (3 marks)

- (b) The diagram shows a pentagon  $CDEFG$ , formed by the rectangle  $CHFG$  and two isosceles triangles  $CDH$  and  $EFH$ , where  $DHE$  is a straight line.

The angle  $EHF$  is  $\theta$ ,  $CD = DH = 3$  cm,  $EH = EF = 4$  cm.



- (i) The perimeter of the pentagon is  $P$  cm.  
 Show that  $P = 14 + 6 \sin \theta + 8 \cos \theta$ . (3 marks)
- (ii) Write down the maximum possible value of  $P$  and find the value of  $\theta$  at which this maximum value occurs. Give the value of  $\theta$  to three decimal places. (3 marks)
- (c) (i) The area of the pentagon is  $A$  cm<sup>2</sup>.  
 Show that  $A = 36.5 \sin 2\theta$ . (5 marks)
- (ii) Hence show that the perimeter of the pentagon with maximum area is  $7(2 + \sqrt{2})$  cm. (2 marks)

**END OF QUESTIONS**

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