Version 1.0: 0805

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6300
Specification A

MAP2 Pure 2

Mark Scheme

2005 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
–x EE		deduct x marks for each error
NMS		no method shown
PI		possibly implied
SCA		substantially correct approach
c		candidate
sf		significant figure(s)
dp		decimal place(s)

Abbreviations used in Marking

MC – x	deducted x marks for mis-copy
MR – x	deducted x marks for mis-read
ISW	ignored subsequent working
BOD	given benefit of doubt
WR	work replaced by candidate
FB	formulae book

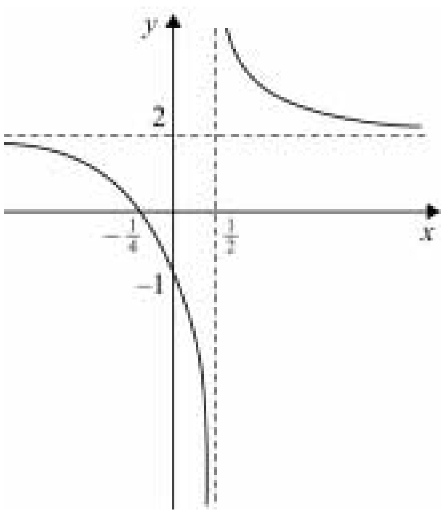
Application of Mark Scheme

No method shown:	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

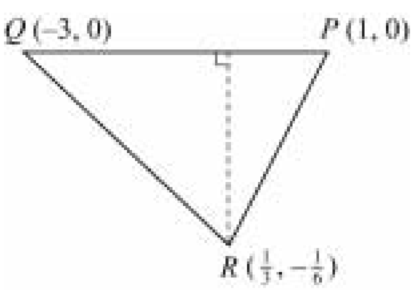
MAP2

Q	Solution	Marks	Total	Comments
1(a)	$(u_1 = 1)$			given
	$u_2 = 1 + \frac{2}{3} \times 3 = 3$	M1		attempt at n = 1, 2, 3, 4 (one or more)
	$u_3 = 3 + \frac{2}{3} \times 3^2 = 9$	A1		any **one** correct answer
	$u_4 = 9 + \frac{2}{3} \times 3^3 = 27$			
	$u_5 = 27 + \frac{2}{3} \times 3^4 = 81$	A1	3	all correct
(b)	A geometric progression $\therefore\ n^{th}$ term: $u_n = 3^{n-1}$	B1	1	
(c)	$S_{100} = \frac{3^{100} - 1}{2}$	M1		use of correct formula
	$= 2.58 \times 10^{47}$	A1	2	AG
	Total		**6**	

MAP2 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\dfrac{4x+1}{2x-1}=\dfrac{2(2x-1)+3}{(2x-1)}$	M1		any valid method
	$=2+\dfrac{3}{(2x-1)}$			
	$\therefore\quad A=2$	A1		for value of A seen anywhere
	and $B=3$	A1	3	for value of B seen anywhere
(b)	Asymptotes are: $x=\dfrac{1}{2}$ and $y=2$	B1B1√	2	
(c)		B1√		for asymptotes
		B1		points of intersection
		B1	3	shape
(d)	$\dfrac{4x+1}{2x-1}<-1$	B1		for $x>0$
	Solution is $0<x<\dfrac{1}{2}$	B1√	2	from vertical asymptote
	Total		**10**	

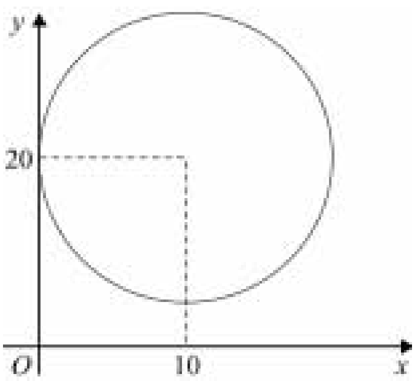
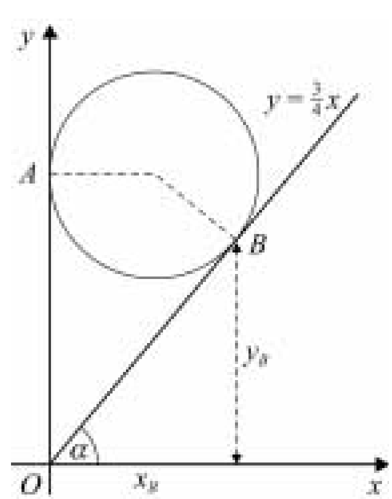
MAP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$y = x^3 - 7x + 6$			
	$\frac{dy}{dx} = 3x^2 - 7$	B1		
	$\left.\frac{dy}{dx}\right\|_{x=1} = -4$			
	$\therefore$ slope of normal at $(1,0)$ is $\frac{1}{4}$	B1√		ft on their $\frac{dy}{dx}$
	Equation of the normal at the point (1, 0) is:			
	$y = \frac{1}{4}(x-1)$	M1		for any correct form $(y - y_1 = m(x - x_1)$ or $y = mx + c)$ seen
	$4y - x + 1 = 0$	A1√	4	AG; ft on their $\left.\frac{dy}{dx}\right\|_{x=1}$
(b)	The point $R(2k, -k)$ lies on this normal			
	$\therefore\ 4(-k) - (2k) + 1 = 0$	M1		
	$6k = 1$			
	$k = \frac{1}{6}$	A1	2	
(c)	Area of $\Delta PQR = \frac{1}{2} \times 4 \times \frac{1}{6}$	M1		
	$= \frac{1}{3}$ square unit	A1√	2	
	Total		**8**	

MAP2 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\sec x = 2 \Rightarrow \cos x = \frac{1}{2}$	M1		
	$x = \frac{\pi}{3}, \frac{5\pi}{3}$	A1	2	AWRT 1.05^c and 5.24^c
(b)	$\cos 2x = \cos(x+x)$ $= \cos x \times \cos x - \sin x \times \sin x$ $= \cos^2 x - \sin^2 x$	M1		
	$= \cos^2 x - (1-\cos^2 x)$ $= 2\cos^2 x - 1$	A1	2	AG
(c)	$(2\cos^2 x - 1) + 3\cos x - 1 = 0$	M1		
	$2\cos^2 x + 3\cos x - 2 = 0$			
	$(2\cos x - 1)(\cos x + 2) = 0$	m1		for factorisation attempted (or use of formula or completing the square)
	$\cos x = -2 \Rightarrow$ no solutions	B1		
		M1		for $\cos x = \frac{1}{2}$
	$\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$	A1√	5	ft on their (a)
	Total		**9**	
5(a)	$\frac{dy}{dx} = \frac{(\sin x)e^x - e^x \cos x}{\sin^2 x}$	M1A1 A1	3	
(b)(i)	When $x = \frac{\pi}{4}$,			
	$\frac{dy}{dx} = \frac{\left(\sin\frac{\pi}{4}\right)e^{\frac{\pi}{4}} - e^{\frac{\pi}{4}}\left(\cos\frac{\pi}{4}\right)}{\sin^2\frac{\pi}{4}}$	M1		stated or attempted
	$= 0 \therefore$ stationary point when $x = \frac{\pi}{4}$	A1	2	
(ii)	When $x = \frac{\pi}{4}$,			
	$y = \left(\frac{e^{\frac{\pi}{4}}}{\sin\frac{\pi}{4}}\right) = \frac{e^{\frac{\pi}{4}}}{\frac{1}{\sqrt{2}}} = \sqrt{2}e^{\frac{\pi}{4}} = 3.10$	M1 A1	2	
	Total		**7**	

MAP2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Radius = 10 Centre = (10, 20)	B1 B1	2	
(b)(i)	 Centre (10, 20) $\Rightarrow$ $OA = 20$ $OB = OA = 20$ (tangents equal length)	B1	1	CAO
(ii)	$\tan\alpha = \frac{3}{4}$ $x_B = 20\times\cos\alpha$ $= 20\times\frac{4}{5}$ $= 16$	M1 A1	2	or use substitution of $y = \frac{3}{4}x$ into the equation of the circle to give $x = 16$ for M1A1
(iii)	$y_B = 20\times\sin\alpha$ $= 20\times\frac{3}{5}$ $= 12$ $d^2 = (16-0)^2 + (12-20)^2$ $d^2 = 256 + 64$ $d^2 = 320$ $d = \sqrt{320} = \sqrt{16\times4\times5}$ $d = 8\sqrt{5}$	B1 M1 A1	3	
	Total		**8**	

MAP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\int_0^\pi x\sin x\,dx$			
	$=\left[x(-\cos x)\right]_0^\pi-\int_0^\pi 1(-\cos x)\,dx$	M1		integration by parts
	$=\left[-x\cos x\right]_0^\pi+\int_0^\pi \cos x\,dx$			
	$=\left[-x\cos x+\sin x\right]_0^\pi$	A1		
		m1		substitution of correct limits
	$=\pi$	A1	4	AG
(b)(i)	$\sin^2 x=\frac{1}{2}(1-\cos 2x)$	B1	1	
(ii)	$\int_0^\pi \sin^2 x\,dx=\frac{1}{2}\int_0^\pi (1-\cos 2x)\,dx$	M1		
	$=\frac{1}{2}\left(x-\frac{\sin 2x}{2}\right)_0^\pi$	A1√		
	$=\frac{\pi}{2}$	A1	3	AG
(c)	$y^2=(2x+\sin x)^2=4x^2+4x\sin x+\sin^2 x$	B1		
	$V=\pi\int_0^\pi (4x^2+4x\sin x+\sin^2 x)\,dx$	M1		$V=\pi\int_0^\pi (2x+\sin x)^2\,dx$
	$=4\pi\int_0^\pi x^2\,dx+4\pi\int_0^\pi x\sin x\,dx+\pi\int_0^\pi \sin^2 x\,dx$			
	$=4\pi\left[\frac{x^3}{3}\right]_0^\pi+(4\pi\times\pi)+\left(\pi\times\frac{\pi}{2}\right)$	M1		$\pi^2\left(\frac{8\pi^2+27}{6}\right)$
	$=174$	A1	4	174.29 (AWRT 174)
	Total		**12**	
	TOTAL		**60**	