GCE 2004

June Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics A

Unit MAP2

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Key to Mark Scheme

M..................................mark is for .... method
m ....mark is dependent on one or more M marks and is for..................... method
A..................................mark is dependent on M or m marks and is for .... accuracy
B..................................mark is independent of M or m marks and is for ....method and accuracy
E..................................mark is for ....explanation
√ or ft or F.. follow through from previous incorrect result
CAO..correct answer only
AWFW ....anything which falls within
AWRT ....anything which rounds to
AG .... answer given
SC .... special case
OE.. or equivalent
A2,1... 2 or 1 (or 0) accuracy marks
-*x* EE...deduct *x* marks for each error
NMS...no method shown
PI ....possibly implied
SCA ....substantially correct approach
c.. candidate
SF...significant figure(s)
DP .... decimal place(s)

Abbreviations used in Marking

MC – *x*..deducted *x* marks for mis-copy
MR – *x*..deducted *x* marks for mis-read
ISW... ignored subsequent working
BOD..given benefit of doubt
WR... work replaced by candidate
FB .... formulae booklet

Application of Mark Scheme

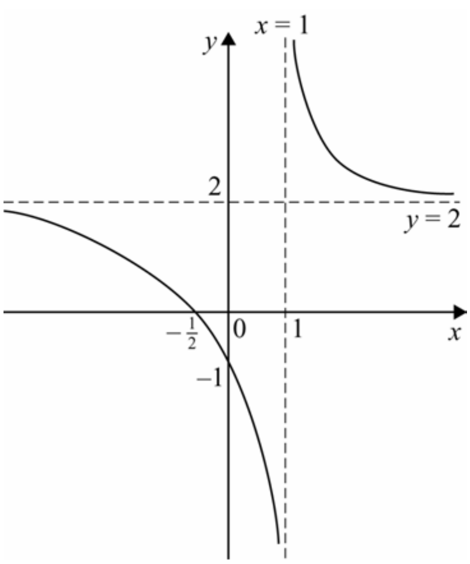
No method shown:
Correct answer without working..mark as in scheme
Incorrect answer without working ....zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

MAP2

Q	Solution	Marks	Total	Comments
1(a)(i)	Crosses y-axis when $x = 0$ i.e. when $y = -1$	B1	1	
(ii)	crosses x-axis when $y = 0$ i.e. when $2x + 1 = 0$, $x = -\frac{1}{2}$	B1	1	
(b)(i)	$\frac{2x+1}{x-1} = \frac{2(x-1)+3}{x-1}$	M1		OE
	$= 2 + \frac{3}{x-1}$	A1A1	3	Accept $A = 2$ & $B = 3$
(ii)	$x = 1$	B1		
	$y = 2$	B1ft	2	
(c)		B3	3	B1 ft asymptotes B1 ft intercepts (on part (a)) B1 shape
(d)	$\frac{2x+1}{x-1} \le 0$ *for* $-\frac{1}{2} \le x < 1$	B1		for $-\frac{1}{2}$ and $\le$
		B1	2	for 1 and $<$ (B1 for end points correct)
	Total		**12**	

MAP2 (Cont)

Q	Solution	Marks	Total	Comments
2(a)	$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$......(i)			
	$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$.......(ii)			
	add the two equations (i) & (ii) together	M1		
	$\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cos\beta$	A1	2	AG
(b)(i)	$2\sin 8x\cos 2x=\sin(8x+2x)+\sin(8x-2x)$	M1		
	$=\sin 10x+\sin 6x$	A1	2	
(ii)	$\int 6\sin 8x\cos 2x\,\mathrm{d}x$			
	$=3\int(\sin 10x+\sin 6x)\,\mathrm{d}x$	M1ft		Use their (i)
	$=3\left(\frac{-\cos 10x}{10}-\frac{\cos 6x}{6}\right)+c$	M1ft		Integration attempted
	$=-\frac{3}{10}\cos 10x-\frac{1}{2}\cos 6x+c$	A1ft	3	Any correct form
	Total		**7**	

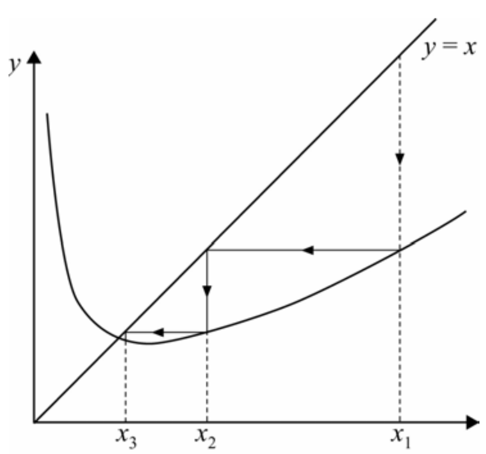
MAP2 (Cont)

Q	Solution	Marks	Total	Comments
3(a)	$\int_0^{\frac{\pi}{2}} x\cos x\,dx$			
	$= x\sin x - \int \sin x\,dx$	M1 M1		
	$= \{x\sin x + \cos x\}_0^{\frac{\pi}{2}}$	A1		
	$= \frac{\pi}{2} - 1$	M1 A1	5	Radians only 0.570 to 0.571
(b)(i)	$t = x^2 + 4 \Rightarrow dt = 2x\,dx$	M1		correct
	$\therefore \int \frac{2x\,dx}{\sqrt{x^2+4}} = \int \frac{dt}{\sqrt{t}}$	A1	2	AG
(ii)	$\int_0^2 \frac{2x\,dx}{\sqrt{x^2+4}} = \int_4^8 t^{-\frac{1}{2}}\,dt$			
	$\left[2\sqrt{t}\right]$ or $\left[2\sqrt{x^2+4}\right]$	M1 A1		Integration attempted correct
	$= 2\sqrt{8} - 2\sqrt{4}$	M1		attempt at correct limits seen
	$= 2(2\sqrt{2}) - 4$			
	$= 4(\sqrt{2} - 1)$	A1	4	AG (AWRT 1.7)
	Total		**11**	

MAP2 (Cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$\frac{dy}{dx} = e^x \times 2\cos 2x + e^x \times \sin 2x$	M1 A1A1	3	Use of product rule A1 for each part correct
(ii)	$\left.\frac{dy}{dx}\right\|_{x=0} = 2$ $\therefore y = mx \Rightarrow$ equation of tangent at (0, 0) is $y = 2x$	M1 A1ft	 2	
(b)	$\left.\frac{dy}{dx}\right\|_{x=\pi} = 2e^{\pi}$ $\therefore$ gradient of normal at $x = \pi$ is $-\frac{1}{2e^{\pi}}$ when $x = \pi$, $y = 0$ $\therefore$ equation of normal at $(\pi, 0)$ is given by $y = -\frac{1}{2e^{\pi}}(x - \pi)$ $\Rightarrow 2e^{\pi} y + x = \pi$	 M1 B1 M1ft A1	 4	 Use of $m_1 \times m_2 = -1$ (– 0.216) on their gradient of normal AG (any correct form)
	Total		**9**	

MAP2 (Cont)

Q	Solution	Marks	Total	Comments
5(a)	$f(x) = x^3 - 15$			
	$f(2) = -7 < 0$	B1		values
	$f(3) = 12 > 0$	E1	2	change of sign
	$\therefore$ root in the interval $[2, 3]$			
(b)(i)	$x = \frac{2}{3}x + \frac{5}{x^2}$			
	$(\times 3x^2) \Rightarrow 3x^3 = 2x^3 + 15$	M1		
	$x^3 - 15 = 0$	A1	2	AG
(ii)	$x_{n+1} = \frac{2}{3}x_n + \frac{5}{x_n^2}$			
	using $x_1 = 3$,	M1		
	$x_2 = 2.555556$	A1		
	$x_3 = 2.469299$	A1√		on their x_2
	$x_4 = 2.466216$	A1√	4	2.466215932
(iii)		B2	2	B1 for staircase B1 for convergence
(iv)	$\sqrt[3]{15}$	B1	1	
	Total		**11**	

MAP2 (Cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$C(4, 3)$	B1		
(ii)	$r = 2$	B1	2	
(b)(i)	$(x-4)^2+(y-3)^2=4$ and $y=x+1$			
	meet when $(x-4)^2+(x+1-3)^2=4$	M1		Substitution attempted or eliminating x
	$\Rightarrow (x-4)^2+(x-2)^2=4$			
	$(x^2-8x+16)+(x^2-4x+4)=4$	M1		Multiply out correctly and simplification attempted
	$2x^2-12x+20=4$			
	$x^2-6x+8=0$	A1		quadratic
	$(x-4)(x-2)=0$	M1		factorise/other valid method attempted
	$x=4$ or $x=2$			
	$x=4 \Rightarrow y=5$ $x=2 \Rightarrow y=3$ $A(4, 5)$ & $B(2, 3)$	A1ft	5	Both points (cao)
(ii)	Area of segment $=\frac{1}{4}\pi(2)^2-\frac{1}{2}(2\times2)$	M1		$\frac{1}{4}\times$circle - triangle
		A1ft		(on their value of r)
	$=\pi-2$	A1	3	AG (AWRT 1.14)
	Total		**10**	
	Total		**60**	