GCE 2005
January Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics A
(MAP2)

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Key to Mark Scheme

M ..... mark is for ..... method
m ..... mark is dependent on one or more M marks and is for ..... method
A ..... mark is dependent on M or m marks and is for ..... accuracy
B ..... mark is independent of M or m marks and is for ..... method and accuracy
E ..... mark is for ..... explanation
√ **or ft or F** ..... follow through from previous incorrect result
CAO ..... correct answer only
AWFW ..... anything which falls within
AWRT ..... anything which rounds to
AG ..... answer given
SC ..... special case
OE ..... or equivalent
A2,1 ..... 2 or 1 (or 0) accuracy marks
-*x* EE ..... deduct *x* marks for each error
NMS ..... no method shown
PI ..... possibly implied
SCA ..... substantially correct approach
c ..... candidate
SF ..... significant figure(s)
DP ..... decimal place(s)

Abbreviations used in Marking

MC – *x* ..... deducted *x* marks for mis-copy
MR – *x* ..... deducted *x* marks for mis-read
ISW ..... ignored subsequent working
BOD ..... given benefit of doubt
WR ..... work replaced by candidate
FB ..... formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working ..... mark as in scheme
Incorrect answer without working ..... zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

Crossed out work — do not mark unless it has not been replaced

Alternative solution using a correct or partially correct method — award method and accuracy marks as appropriate

MAP1

Q	Solution	Marks	Total	Comments
1(a)	Formula for *n*th term of AP	M1		Stated or used
	$n = \frac{1}{3}(800-101)+1 = 234$	A1	2	Shown, not verified (AG)
(b)	Formula for sum of AP	M1		Stated or used
	$S = \frac{234}{2}(101+800)$			
	or $S = \frac{234}{2}(2(101)+3(233))$	m1		Allow one error here
	$\ldots = 105\,417$	A1	3	
(c)	$S = \frac{117}{2}(104+800)$			
	Or $S = \frac{117}{2}(2(104) + 6\,(116))$	M1		Allow one error here
	…= 52 884	A1	2	
	Total		**7**	
2(a)(i)	$y' = 4\ldots$	B1		
	$\ldots - 9x^{-2}$	M1A1	3	M1 for kx^{-2}
(ii)	At SP $4 = 9x^{-2}$	M1		
	$\Rightarrow x^2 = \frac{9}{4}$	m1		OE
	SPs are $(\frac{3}{2}, 12)$...	A1A1		
	…and $\left(-\frac{3}{2}, -12\right)$	A1	5	
(b)(i)	$\int y\,dx = 2x^2 + 9\ln x\ (+c)$	M1A1	2	M1 if one term correct
(ii)	Substitutions and subtraction	M1		F(2) – F(1) in c's F(x) (not in y or y')
	Area = (8 + 9 ln 2) – 2	m1		Condone one small error, e.g. use of
	= 6 + 9 ln 2	A1	3	decimals
	Total		**13**	

MAP2

Q	Solution	Marks	Total	Comments
1(a)(i)	$\alpha + \beta = 3$	B1	1	
(ii)	$\alpha\beta = 9$	B1	1	
(b)(i)	$\frac{6}{\alpha} \times \frac{6}{\beta} = \frac{36}{\alpha\beta} = 4$	B1ft	1	
(ii)	$\frac{6}{\alpha} + \frac{6}{\beta} = \frac{6(\alpha+\beta)}{\alpha\beta} = 2$	M1A1ft	2	
(c)	New quadratic equation is:			
	$x^2 - 2x + 4$	M1A1√	2	On their b(i) and b(ii)
	Total		**7**	
2(a)	$f(x) = xe^x - 5 = 0$			
	$f(1) = e - 5 < 0$			(– 2.28)
	$f(2) = 2e^2 - 5 > 0$	B1		(9.78)
	Change of sign $\Rightarrow$ root in range $1 < x < 2$	E1	2	
(b)	$\frac{d}{dx}(xe^x) = xe^x + e^x$	M1A1	2	
(c)	$x_0 = 1.2$			
	$f(1.2) = -1.01586$			
	$f'(1.2) = 7.30426$	B1		(can be implied by what follows)
	$x_1 = 1.2 - \left\{\frac{-1.01586}{7.30426}\right\}$	M1		
	$x_1 = 1.2 + 0.13908$			Evidence of use of Newton-Raphson
	$x_1 = 1.339$ (3dp)	A1	3	(on their $f'(1.2)$)
	Total		**7**	

MAP2 (cont)

Q	Solution	Marks	Total	Comments
3	$f(x) = x^3 + ax^2 + bx + 6$			
	$f(1) = 1 + a + b + 6 = 24a + b = 17$	M1		Substitution of 1 or – 2 attempted.
	$f(-2) = -8 + 4a - 2b + 6 = 24$			
	$\Rightarrow 4a - 2b = 26 \Rightarrow 2a - b = 13$	A1		Correct equations
	$\Rightarrow a = 10$ and $b = 7$	A1√A1 √	4	ft on their equations
	Total		**4**	
4(a)(i)	$\frac{d}{dx}\left(\ln\left[1 + x^2\right]\right) = \frac{2x}{1 + x^2}$	M1A1	2	
(ii)	$\int_0^1 \frac{x}{1 + x^2} dx \left[= \frac{1}{2}\ln\left(1 + x^2\right)\right]_0^1$	M1		
	$= \frac{1}{2}\ln 2 - \frac{1}{2}\ln 1$			
	$= \frac{1}{2}\ln 2$	A1	2	(0.347)
(b)(i)	$y = \tan^{-1} x \Rightarrow x = \tan y$	B1	1	
(ii)	$\frac{dx}{dy} = \sec^2 y$	B1	1	OE
(iii)	$\sec^2 y = 1 + \tan^2 y$	M1		
	$= 1 + x^2$			
	$\therefore \frac{dy}{dx} = \frac{1}{\sec^2 y}$			
	$= \frac{1}{1 + x^2}$	A1	2	
(iv)	$\int_0^1 \frac{dx}{1 + x^2} = \left[\tan^{-1} x\right]_0^1$	M1		
	$= \frac{\pi}{4}$	A1	2	(0.785^c)
(c)	Shaded area $= \frac{\pi}{4} - \frac{1}{2}\ln 2$	M1A1	2	(on their b(iv) and a(ii))
	Total		**12**	

MAP2 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$x = 0 \quad y = 2\sqrt{3} = 3.4641$			
	$x = 1 \quad y = \sqrt{15} = 3.8730$			
	$x = 2 \quad y = 4$			
	$x = 3 \quad y = \sqrt{15} = 3.8730$	M1		For correct x-values attempted
	$x = 4 \quad y = 2\sqrt{3} = 3.4641$			
	$x = 5 \quad y = \sqrt{7} = 2.6458$			
	$x = 6 \quad y = 0$	A1		
	Area $= \frac{1}{2} \times 1 \times \{2\sqrt{3} + 0 + 2(17.8558)\}$	M1		
	Area $= \frac{1}{2} \times 39.176$			
	Area = 19.6	A1	4	(AWRT 19.6)
(b)(i)	Radius of circle = 4	B1	1	[6 – 2 = 4; $OB - OC = r$]
(ii)	In ΔACO $\cos ACO = \frac{2}{4} = 0.5$	M1		
	$\text{ACO} = 60^\circ$			
	$ACB = 180^\circ - 60^\circ$			
	$= 120^\circ$	A1	2	

MAP2 (cont)

Q	Solution	Marks	Total	Comments
5(c)(i)	sector $ACB = \frac{1}{2}\times 4^2 \times \frac{2\pi}{3}$			
	$= \frac{16\pi}{3}$	B1	1	(16.8)
(ii)	Shaded area = $\Delta\, AOC$ + sector CAB	M1		Δ attempted
	$\Delta\, AOC = \frac{1}{2}\times 2\times 2\sqrt{3}$			
	$= 2\sqrt{3}$			
	Exact value of shaded area is:			
	$\frac{16\pi}{3} + 2\sqrt{3}$	A1	2	AG
(d)	Volume = $\pi \int_0^6 y^2 \mathrm{d}x$	M1		Correct integration attempted.
	Volume = $\pi \int_0^6 \left[16 - (x-2)^2\right] \mathrm{d}x$			
	$= \left[16\pi x\right]_0^6 - \pi\left[\frac{1}{3}(x-2)^3\right]_0^6$	A1A1		Correct integrations.
	$= (96 - 24)\,\pi$			
	$= 72\,\pi$	A1	4	CAO (226)
	Total		**14**	

MAP2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$6\sin\theta+8\cos\theta \equiv R\sin(\theta+\alpha)$			
	$\equiv R\sin\theta\cos\alpha+R\cos\theta\sin\alpha$			
	$\Rightarrow R\sin\alpha=8$			
	$R\cos\alpha=6$			
	$\tan\alpha=\frac{4}{3}$	M1A1		
	$\alpha=0.927^c$			AWRT 0.927 or 53. 13°
	and $R=10$	B1	3	
	$6\sin\theta+8\cos\theta \equiv 10\sin(\theta+0.927^c)$			
(b)(i)	$CG=2\times4\cos\theta=8\cos\theta$	B1		
	$GF=2\times3\sin\theta=6\sin\theta$	B1		
	Perimeter $=3+3+4+4+GF+CG$			
	$=14+6\sin\theta+8\cos\theta$	B1	3	
(ii)	$P=14+10\sin(\theta+\alpha)$			
	$P_{max}=24$	B1√		(on their R from (a))
	When $\sin(\theta+\alpha)=1$			
	$\Rightarrow\theta+\alpha=\frac{\pi}{2}$	M1		
	$\theta=\frac{\pi}{2}-0.9273$			(36.9°)
	$\theta=0.644^c$ (3dp)	A1√	3	(on their α from (a))

MAP2 (cont)

Q	Solution	Marks	Total	Comments
6(c)(i)	$\Delta CDH = \frac{1}{2}\times 3\times 3\times \sin 2\theta = 4.5\sin 2\theta$	M1		
	$\Delta EFH = \frac{1}{2}\times 4\times 4\times \sin(\pi - 2\theta)$	A1		
	$= 8\sin 2\theta$			
	$\Delta CHFG = 8\cos\theta \times 6\sin\theta$	M1		
	$= 24\sin 2\theta$	A1		
	Total area of the pentagon is given by:			
	$A = 36.5\sin 2\theta$	A1	5	AG
(ii)	$A_{max} = 36.5$ when $\sin\theta = 1$			
	$\Rightarrow \theta = \frac{\pi}{4}$	M1		$\theta = 45^\circ$
	$\therefore P\left(\theta = \frac{\pi}{4}\right) = 14 + 6\sin\frac{\pi}{4} + 8\cos\frac{\pi}{4}$			
	$= 14 + 14\times\frac{\sqrt{2}}{2}$			
	$= 14 + 7\sqrt{2}$			
	$= 7\left(2+\sqrt{2}\right)(\text{cm})$	A1	2	
	Total		**16**	
	Total		**60**	