General Certificate of Education
June 2005
Advanced Subsidiary Examination

MATHEMATICS (SPECIFICATION A) MAP1
Unit Pure 1

Tuesday 7 June 2005 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Show that the equation

$$x^3 + x^2 - 6 = 0$$

has a root between 1.5 and 1.6. *(3 marks)*

2 The first three terms of a geometric sequence are

$$12, \quad 48, \quad 192$$

(a) Write down the common ratio. *(1 mark)*

(b) Find an expression for the nth term. *(2 marks)*

(c) Show that the sum of the first 30 terms is $2^{62} - 4$. *(4 marks)*

3 A curve has equation

$$y = 4x + x^{-\frac{1}{2}}, \quad x > 0.$$

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. *(5 marks)*

(b) Verify that the curve has a stationary point when $x = \frac{1}{4}$. *(2 marks)*

(c) (i) Find the value of $\frac{d^2y}{dx^2}$ at the stationary point. *(1 mark)*

(ii) Determine whether this point is a maximum or a minimum. *(1 mark)*

4 A building has a large sloping tiled roof. The tiles are arranged in horizontal rows. The numbers of tiles in the rows form an arithmetic sequence.

In the top row there are 120 tiles.

Below that, each row has 8 tiles more than the row above it.

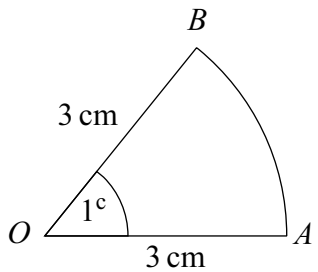
The bottom row has 280 tiles.

Calculate:

(a) the number of rows of tiles on the roof; *(2 marks)*

(b) the number of tiles on the roof. *(3 marks)*

5 The following diagram shows a sector OAB of a circle with centre O and radius 3 cm. The angle AOB is 1 radian.

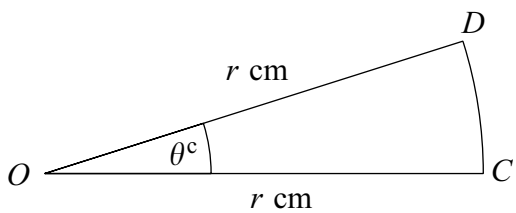


(a) Find:

(i) the perimeter of the sector OAB; *(2 marks)*

(ii) the area of the sector OAB. *(2 marks)*

(b) The sector OCD has radius r cm and angle θ radians, as shown below.



(i) The area of the sector OCD is equal to the area of the sector OAB. Show that

$$\theta = \frac{9}{r^2}.$$ *(2 marks)*

(ii) The perimeter of the sector OCD is twice the perimeter of the sector OAB. Show that

$$2r + \frac{9}{r} = 18.$$ *(2 marks)*

(iii) Deduce that

$$2r^2 - 18r + 9 = 0.$$ *(2 marks)*

(iv) Given that $r > 3$, find the value of r and the corresponding value of θ. Give each answer to three significant figures. *(4 marks)*

6 The angle x, measured in radians, satisfies the equation

$$2\sin^2 x = 1 + \cos x\,.$$

(a) Verify that one root of this equation is $\frac{\pi}{3}$. *(2 marks)*

(b) Use a trigonometric identity to show that

$$2\cos^2 x + \cos x - 1 = 0\,.$$ *(2 marks)*

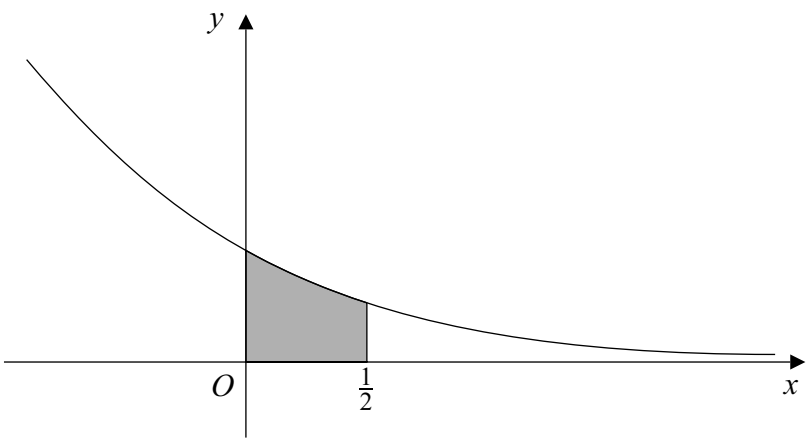
(c) Hence find all the roots of the equation

$$2\sin^2 x = 1 + \cos x$$

in the interval $0 \leqslant x < 2\pi$. *(4 marks)*

7 The diagram shows the graph of

$$y = \mathrm{e}^{-2x}.$$



(a) Describe a sequence of **two** transformations by which the graph of $y = \mathrm{e}^{x}$ can be transformed into that of $y = \mathrm{e}^{-2x}$. *(4 marks)*

(b) (i) Find $\int \mathrm{e}^{-2x}\,\mathrm{d}x$. *(2 marks)*

(ii) Hence show that the shaded region on the diagram has area

$$\frac{1}{2}\left(1 - \frac{1}{\mathrm{e}}\right).$$ *(2 marks)*

(c) The function f is defined for all real numbers by

$$\mathrm{f}(x) = \mathrm{e}^{-2x}.$$

(i) Find the range of the function f. *(1 mark)*

(ii) Find an expression for $\mathrm{f}^{-1}(x)$. *(3 marks)*

(iii) Show that

$$\mathrm{e}^{-2} \leqslant x \leqslant \mathrm{e}^{2} \Rightarrow |\,\mathrm{f}^{-1}(x)\,| \leqslant 1\,.$$ *(2 marks)*

END OF QUESTIONS

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