

General Certificate of Education
June 2004
Advanced Subsidiary Examination



MATHEMATICS (SPECIFICATION A)
Unit Pure 1

MAP1

Wednesday 9 June 2004 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- a ruler;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 (a) Find the sum of the 100 terms of the arithmetic series

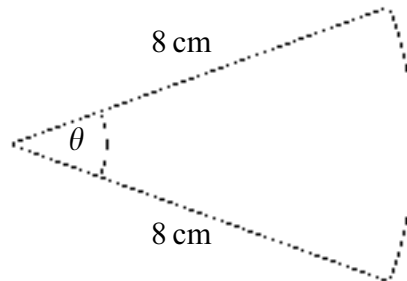
$$3 + 7 + 11 + \dots + 399. \quad (3 \text{ marks})$$

- (b) An arithmetic sequence u_1, u_2, u_3, \dots has r th term u_r , where

$$u_r = 8r - 2.$$

- (i) Write down the values of u_1, u_2, u_3 and u_4 . (2 marks)
- (ii) Using your answer to part (a), or otherwise, find the sum of the first 100 terms of this sequence. (2 marks)

- 2 The diagram shows a sector of a circle of radius 8 cm. The sector has angle θ radians. The perimeter of the sector is P cm and its area is A cm².



- (a) Show that $P = 8(\theta + 2)$. (2 marks)
- (b) Find A in terms of θ . (2 marks)
- (c) Given that $A = P$, find the value of θ . (3 marks)

- 3 (a) Show that the equation

$$2x^{\frac{3}{2}} - 9x + 6 = 0$$

has a root between 0 and 1.

(3 marks)

- (b) A curve has equation

$$y = 2x^{\frac{3}{2}} - 9x.$$

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(5 marks)

- (ii) Calculate the coordinates of the stationary point on the curve.

(3 marks)

- (iii) Find the value of $\frac{d^2y}{dx^2}$ at the stationary point and hence determine whether this point is a maximum or a minimum.

(2 marks)

- 4 Write each of the following in the form $a \ln p + b \ln q$:

(a) $\ln(pq)$;

(1 mark)

(b) $\ln(p^2q^3)$;

(1 mark)

(c) $\ln\left(\frac{p}{q}\right)$;

(1 mark)

(d) $\ln\sqrt{\frac{p}{q}}$.

(1 mark)

- 5 (a) A geometric sequence has first term 230 and second term 345.

- (i) Show that the common ratio is 1.5.

(1 mark)

- (ii) Calculate the third and fourth terms.

(2 marks)

- (b) In 1501 the population of a country was 2 300 000.

In 1601 the population was 3 450 000.

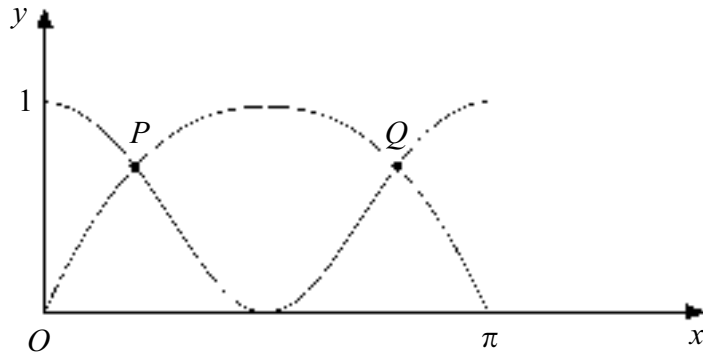
Assuming that the population in the years 1501, 1601, 1701 and 1801 can be modelled as a geometric sequence, write down an estimate for the population in 1801. Give your answer to three significant figures.

(2 marks)

6 The diagram shows the graphs of

$$y = \cos^2 x \text{ and } y = \sin x \text{ for } 0 \leq x \leq \pi.$$

The graphs intersect each other at two points P and Q .



(a) Use a trigonometric identity to show that the x -coordinates of P and Q satisfy the equation

$$\sin^2 x + \sin x - 1 = 0. \quad (2 \text{ marks})$$

(b) (i) Solve this quadratic equation. (2 marks)

(ii) Show that the only possible value for $\sin x$ is approximately 0.618. (2 marks)

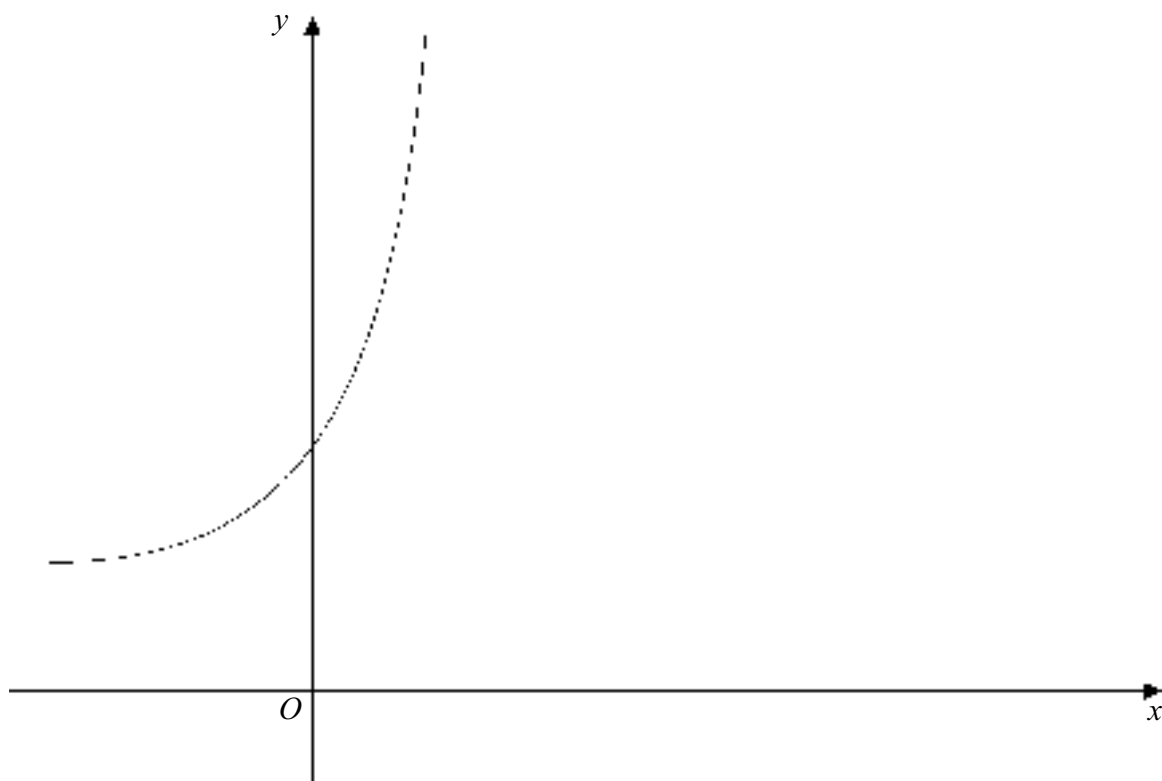
(c) Find the x -coordinates of P and Q , giving each answer to two decimal places. (3 marks)

7 (a) (i) Find $\int (e^{2x} + 1) dx$. (3 marks)

(ii) Hence show that $\int_0^{\ln 2} (e^{2x} + 1) dx = \frac{3}{2} + \ln 2$. (3 marks)

(b) The diagram shows the graph of

$$y = e^{2x} + 1.$$



Find the y -coordinate of the point where the graph intersects:

(i) the y -axis; (1 mark)

(ii) the line $x = \ln 2$. (2 marks)

(c) The function f is defined **on the restricted domain** $0 \leq x \leq \ln 2$ by

$$f(x) = e^{2x} + 1.$$

(i) Find the range of the function f . (1 mark)

(ii) On one pair of axes sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$. (2 marks)

(iii) Find an expression for $f^{-1}(x)$. (3 marks)

END OF QUESTIONS

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