

General Certificate of Education  
January 2005  
Advanced Subsidiary Examination



**MATHEMATICS (SPECIFICATION A)**  
**Unit Pure 1**

**MAP1**

Friday 14 January 2005 Morning Session

**In addition to this paper you will require:**

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

**Information**

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

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1 Consider the arithmetic series

$$101 + 104 + 107 + 110 + \dots + 800.$$

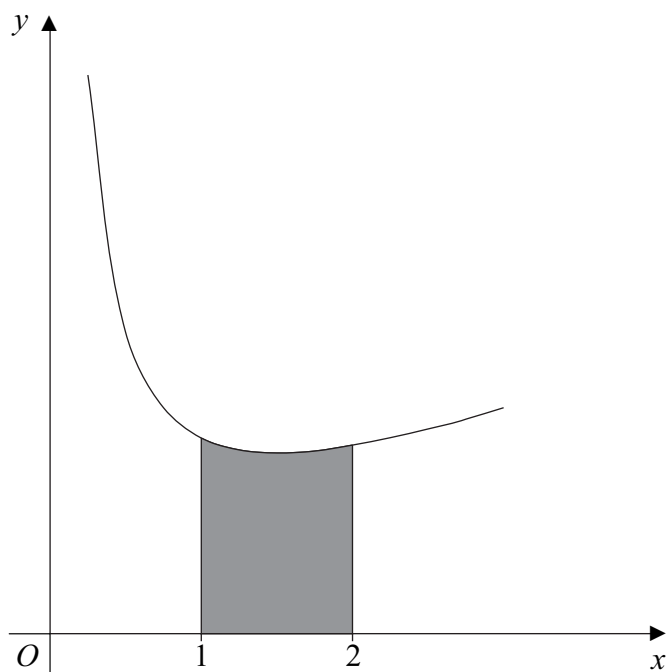
- (a) Show that there are 234 terms in this series. (2 marks)
- (b) Find the sum of the arithmetic series. (3 marks)
- (c) Find the sum of the **even** numbers in the series. (2 marks)

2 A curve has equation

$$y = 4x + 9x^{-1}.$$

- (a) (i) Find  $\frac{dy}{dx}$ . (3 marks)
- (ii) Hence find the coordinates of the **two** stationary points on the curve. (5 marks)
- (b) The diagram shows the graph of

$$y = 4x + \frac{9}{x} \quad \text{for } x > 0.$$



- (i) Find  $\int \left(4x + \frac{9}{x}\right) dx$ . (2 marks)
- (ii) Hence calculate the area of the region shaded on the diagram. Give your answer in the form  $p + q \ln 2$ . (3 marks)

3 (a) Write down the values of  $\tan \frac{\pi}{4}$  and  $\tan \frac{3\pi}{4}$ . (2 marks)

(b) It is given that  $x$  satisfies the equation

$$2 \sin^2 x + \sin x \cos x = \cos^2 x.$$

(i) By dividing both sides of this equation by  $\cos^2 x$ , show that

$$2 \tan^2 x + \tan x - 1 = 0. \quad (2 \text{ marks})$$

(ii) Solve this quadratic equation for  $\tan x$ . (2 marks)

(iii) Hence find all possible values of  $x$  in the interval  $0 < x < \pi$ . Give each answer to three significant figures. (2 marks)

4 (a) Sketch the graph of  $y = \ln x$ , showing the coordinates of the point where the graph intersects the  $x$ -axis. (2 marks)

(b) For the graph of  $y = 1 + \ln x$ , find:

(i)  $\frac{dy}{dx}$ ; (1 mark)

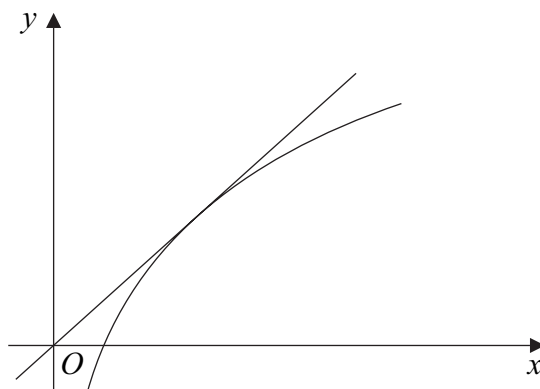
(ii) the gradient of the graph at the point where  $x = 1$ . (1 mark)

(c) The function  $f$  is defined for  $x > 0$  by

$$f(x) = 1 + \ln x.$$

The diagram shows the graphs of

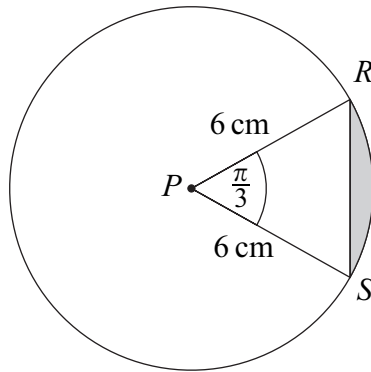
$$y = x \text{ and } y = f(x).$$



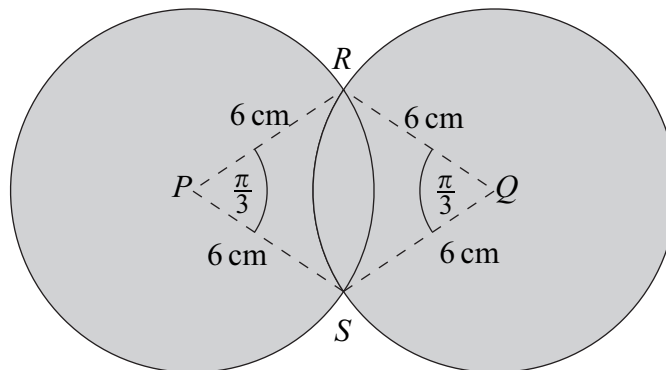
(i) Copy the diagram and, using the same axes, sketch the graph of  $y = f^{-1}(x)$ . (2 marks)

(ii) Find an expression for  $f^{-1}(x)$ . (3 marks)

- 5 The diagram shows a circle with centre  $P$  and radius 6 cm. The angle  $SPR$  is  $\frac{\pi}{3}$  radians.



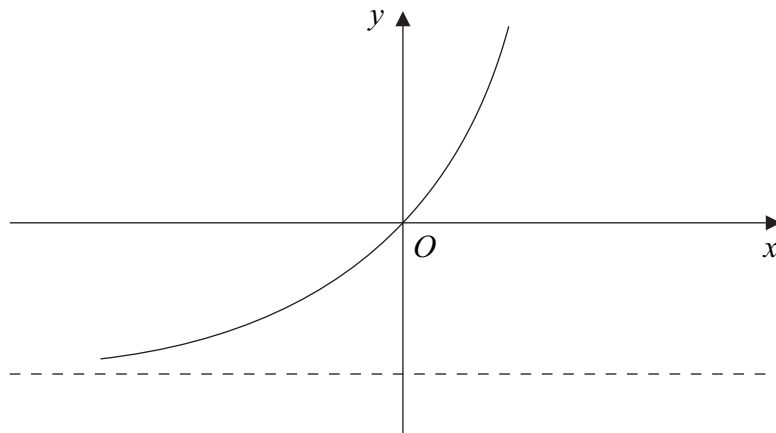
- (a) Find, in terms of  $\pi$ , the length of the arc  $RS$ . (2 marks)
- (b) Find the area of:
- (i) the sector  $SPR$ ; (2 marks)
  - (ii) the triangle  $SPR$ ; (2 marks)
  - (iii) the shaded segment. (1 mark)
- (c) The diagram below shows a logo consisting of two overlapping circles with centres  $P$  and  $Q$  and both with radius 6 cm. The circles intersect at  $R$  and  $S$ , and the angle  $SPR$  is  $\frac{\pi}{3}$  radians.



Show that the area of the front face of the logo is approximately  $220 \text{ cm}^2$ . (3 marks)

- 6 The diagram shows the graph of  $y = f(x)$ , where the function  $f$  is defined for all values of  $x$  by

$$f(x) = e^{2x} - 1.$$



- (a) (i) Describe a sequence of **two** transformations by which the graph of  $y = e^{2x} - 1$  can be obtained from that of  $y = e^x$ . *(4 marks)*
- (ii) Write down the range of the function  $f$ . *(1 mark)*
- (b) For the graph of  $y = e^{2x} - 1$ , find:
- (i)  $\frac{dy}{dx}$ ; *(2 marks)*
- (ii)  $\frac{d^2y}{dx^2}$ . *(1 mark)*
- (c) The function  $g$  is defined for all values of  $x$  by
- $$g(x) = |x|.$$
- (i) Write down an expression for  $gf(x)$ . *(1 mark)*
- (ii) Sketch the graph of  $y = gf(x)$ . *(2 marks)*
- (iii) Show that  $gf(x) > 1 \Rightarrow f(x) > 1$ . *(2 marks)*

**END OF QUESTIONS**

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