

General Certificate of Education
January 2004
Advanced Subsidiary Examination



MATHEMATICS (SPECIFICATION A)
Unit Pure 1

MAP1

Wednesday 14 January 2004 Morning Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Find $\int x^{\frac{1}{2}} dx$. (2 marks)

(b) Hence find the value of $\int_0^2 x^{\frac{1}{2}} dx$,

giving your answer in the form $p\sqrt{2}$, where p is a rational number. (3 marks)

2 The n th term of a geometric sequence is u_n , where

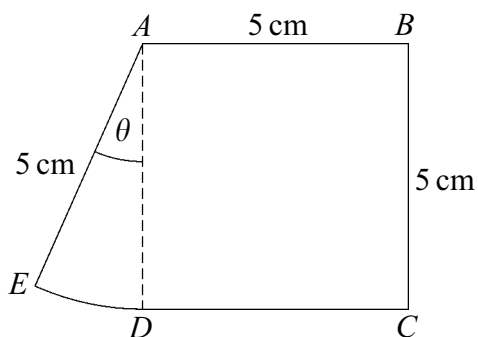
$$u_n = 2 \times 3^n.$$

(a) Find the values of u_1 and u_2 . (2 marks)

(b) Write down the common ratio of the geometric sequence. (1 mark)

(c) Show that the sum of the first 10 terms is $3(3^{10} - 1)$. (3 marks)

3 The diagram shows a shape $ABCDE$. The shape consists of a square $ABCD$, with sides of length 5 cm, and a sector ADE of a circle with centre A and radius 5 cm. The angle of the sector is θ radians.



(a) Find the area of the sector ADE in terms of θ . (2 marks)

(b) The area of the sector ADE is a quarter of the area of the square $ABCD$.

(i) Find the value of θ . (2 marks)

(ii) Find the perimeter of the shape $ABCDE$. (2 marks)

4 (a) An arithmetic sequence has first term 100 and common difference 2.

(i) Write down the second and third terms. (2 marks)

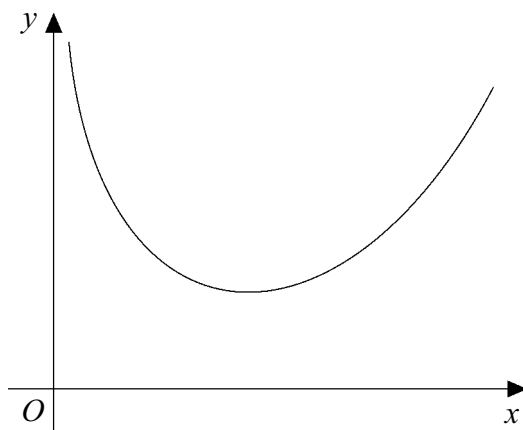
(ii) Given that the last term is 200, find the number of terms. (3 marks)

(b) A tape dispenser has a length of tape wrapped round a circular cylinder. The length of tape in the first layer (nearest to the cylinder) is 100 mm. Each further layer is 2 mm longer than the one before. The outer layer has 200 mm of tape.

Calculate the total length of tape. (3 marks)

5 The diagram shows a sketch of the graph of

$$y = e^{2x} + 2x^{-1} \quad \text{for } x > 0.$$



(a) Find $\frac{dy}{dx}$. (3 marks)

(b) Show that, at the stationary point on the graph, $x^2 e^{2x} = 1$. (3 marks)

(c) Deduce that, at the stationary point,

$$x e^x = 1$$

and hence

$$\ln x + x = 0. \quad \text{(3 marks)}$$

(d) Show that the equation

$$\ln x + x = 0$$

has a root between 0.5 and 0.6. (3 marks)

(e) Find $\int (e^{2x} + 2x^{-1}) dx$. (3 marks)

- 6 (a) The functions f and g are defined by:

$$f(x) = \sqrt{x} \quad \text{for } x \geq 0;$$

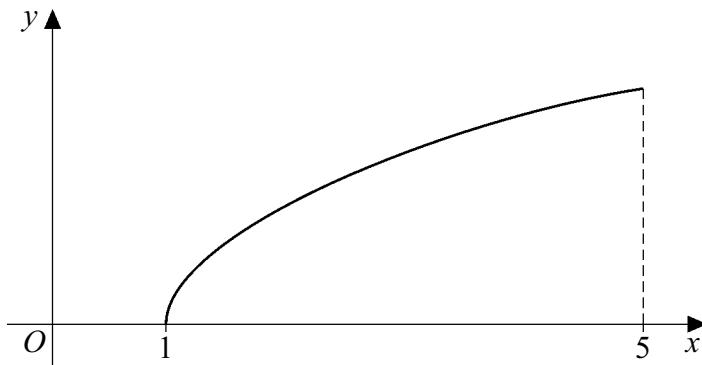
$$g(x) = x - 1 \quad \text{for all values of } x.$$

- (i) Write down expressions for $fg(x)$ and $gf(x)$. (2 marks)
- (ii) Verify that

$$x = 1 \Rightarrow fg(x) = gf(x). \quad (1 \text{ mark})$$

- (b) The diagram shows the graph of $y = h(x)$, where the function h is defined for the domain $1 \leq x \leq 5$ by

$$h(x) = \sqrt{x - 1}.$$



- (i) Describe the transformation by which the graph of $y = \sqrt{x - 1}$ can be obtained from the graph of $y = \sqrt{x}$. (2 marks)
- (ii) Write down the range of the function h . (1 mark)
- (iii) Write down the domain and range of the inverse function h^{-1} . (2 marks)
- (iv) Find an expression for $h^{-1}(x)$. (3 marks)
- 7 (a) Write down the exact values of $\sin \frac{\pi}{6}$, $\cos \frac{\pi}{6}$ and $\tan \frac{\pi}{6}$. (3 marks)
- (b) It is given that x satisfies the equation

$$3 \sin^2 x = \cos^2 x.$$

By first using an appropriate trigonometrical identity to simplify this equation, find all the solutions of the equation in the interval $0 \leq x \leq 2\pi$. (6 marks)

END OF QUESTIONS