

General Certificate of Education
January 2004
Advanced Subsidiary Examination



MATHEMATICS (SPECIFICATION A)
Unit Methods

MAME

Thursday 8 January 2004 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- one sheet of graph paper for use in Question 2;
- a ruler;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAME.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.
- Additional sheets of graph paper are available on request.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 The probability distribution of a random variable X is given in the table below.

| | | | | | |
|------------|-----|-----|-----|-----|-----|
| x | -2 | -1 | 0 | 1 | 2 |
| $P(X = x)$ | 0.1 | 0.1 | 0.2 | 0.3 | 0.3 |

Calculate:

- (a) the mean of X ; *(2 marks)*
- (b) the variance of X . *(3 marks)*

- 2 [A sheet of 2 mm graph paper is provided for use in this question.]

A survey was made of the number of items bought by each of 30 customers at a supermarket. The results are shown in the following stem and leaf diagram.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 4 | 7 | 8 | | |
| 2 | 0 | 1 | 3 | 3 | 7 | 7 |
| 3 | 2 | 5 | 5 | 8 | | |
| 4 | 1 | 3 | 5 | 6 | 8 | 9 |
| 5 | 0 | 4 | 4 | 4 | 9 | |
| 6 | 3 | 6 | 8 | | | |
| 7 | 2 | 5 | | | | |

KEY: 1 | 2 represents 12 items.

- (a) Find the median and quartiles of the distribution. *(4 marks)*
- (b) Draw, on the graph paper provided, a box and whisker diagram to illustrate the distribution. *(4 marks)*

3 It is given that

$$f(x) = x^3 + 4x^2 - 3x - 18.$$

- (a) Find the value of $f(2)$. *(1 mark)*
- (b) Use the Factor Theorem to write down a factor of $f(x)$. *(1 mark)*
- (c) Hence express $f(x)$ as a product of three linear factors. *(4 marks)*

4 Measurements were made of the lengths, x miles, of 20 roads connecting towns in a certain region. The results were summarised as follows:

$$\Sigma x = 320, \quad \Sigma x^2 = 5300.$$

- (a) (i) Calculate the mean of the lengths of the roads in miles. *(1 mark)*
- (ii) Show that the standard deviation of the lengths of the roads is 3 miles. *(2 marks)*
- (b) The lengths were then converted to kilometres using the formula

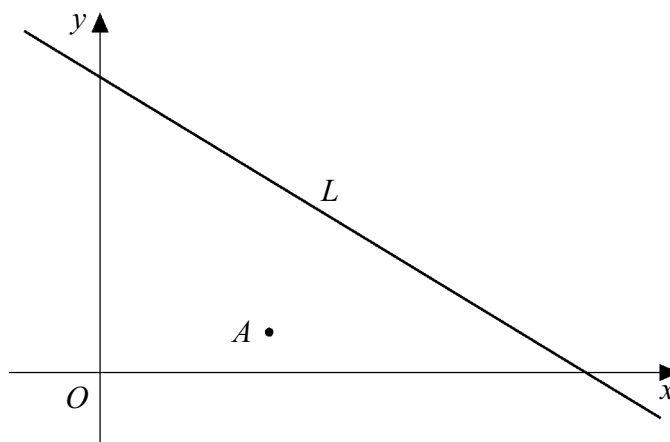
$$y = 1.6x,$$

where y kilometres is equivalent to x miles.

Calculate the mean and the standard deviation of the lengths of the roads in kilometres. *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

- 5 The diagram shows a line L which represents a pipeline, and a point A which is to be connected to the pipeline by the shortest possible connection.



The equation of the line L is

$$2x + 3y = 24,$$

and A is the point $(4, 1)$.

- (a) Find the gradient of the line L . *(2 marks)*
- (b) Hence write down the gradient of a line perpendicular to L . *(1 mark)*
- (c) Show that the line through A perpendicular to L has equation
- $$3x - 2y = 10. \quad \text{span style="float: right;">*(2 marks)*$$
- (d) Hence calculate the coordinates of the point of intersection of the two lines. *(3 marks)*
- (e) Find the length of the shortest possible connection from A to the pipeline. *(2 marks)*

6 A customer goes into a store to buy a refrigerator and a microwave. From past experience it is known that 10% of the refrigerators and 5% of the microwaves will be found to be defective when tested. The customer chooses one refrigerator and one microwave at random and the items are tested.

(a) Find the probability that:

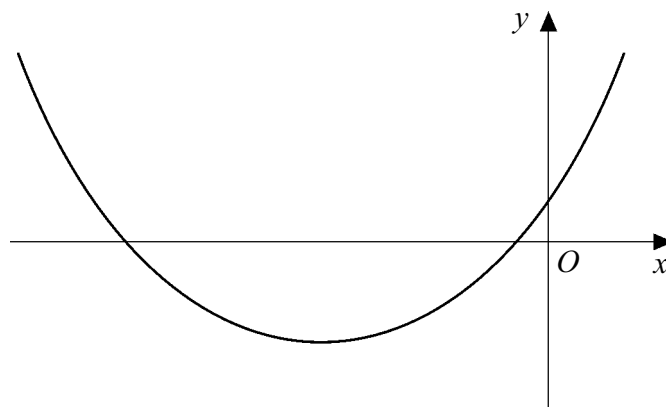
(i) both items are found to be defective; (2 marks)

(ii) neither item is found to be defective; (2 marks)

(iii) exactly one of the items is found to be defective. (2 marks)

(b) Given that exactly one of the items is found to be defective, find the probability that it is the refrigerator. (3 marks)

7 The diagram shows the graph of $y = f(x)$, where $f(x) = x^2 + 6x + 1$.



(a) Express $f(x)$ in the form $(x + m)^2 + n$, where m and n are integers. (2 marks)

(b) Solve the equation $f(x) = 0$, giving your answers in the form $p + q\sqrt{2}$, where p and q are integers. (3 marks)

(c) Solve the inequality $f(x) < 0$. (1 mark)

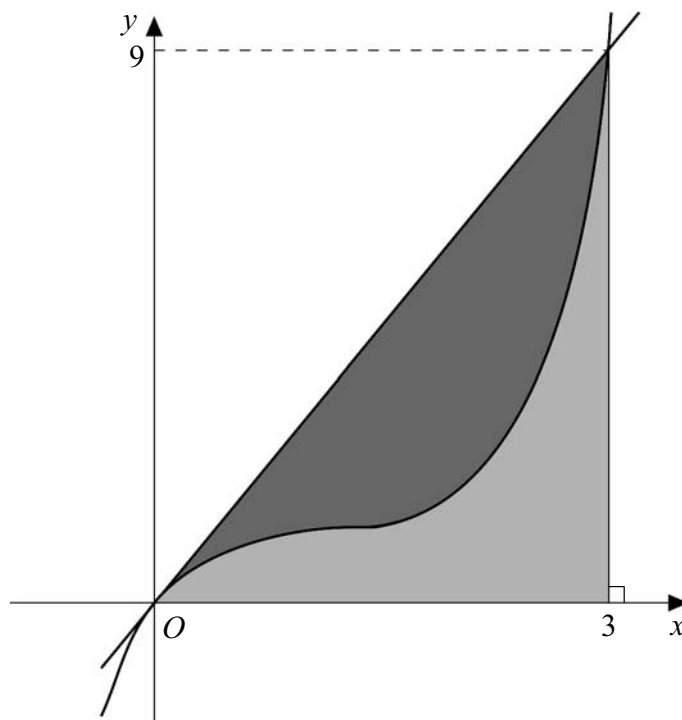
TURN OVER FOR THE NEXT QUESTION

8 The diagram shows the straight line

$$y = 3x$$

and the curve

$$y = x^3 - 3x^2 + 3x.$$



(a) (i) Differentiate $x^3 - 3x^2 + 3x$. (2 marks)

(ii) Find the coordinates of the stationary point on the curve

$$y = x^3 - 3x^2 + 3x. \quad (3 \text{ marks})$$

(b) (i) Find $\int (x^3 - 3x^2 + 3x) dx$. (3 marks)

(ii) Show that the areas of the two shaded regions are equal. (3 marks)

END OF QUESTIONS