General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAM4/W**
Unit Mechanics 4

Tuesday 28 June 2005 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAM4/W.
- Answer **all** questions.
- Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise stated.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A bag of sand of mass 9 kg is dropped from a stationary hot-air balloon. As the sand bag falls through the air, it is subjected to air resistance of magnitude $0.01v^2$ N, where $v\,\mathrm{m\,s^{-1}}$ is the speed of the sand bag at time t seconds.

(a) (i) Write down a differential equation connecting v and t. *(2 marks)*

(ii) Hence find the terminal velocity of the sand bag. *(2 marks)*

(b) Show that, when the speed of the sand bag is $v\,\mathrm{m\,s^{-1}}$, it has fallen a distance of

$$450 \ln \frac{9g}{9g - 0.01v^2} \text{ metres}$$

from the hot-air balloon. *(6 marks)*

2 [In this question take Newton's gravitational constant to be $6.7 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$.]

A projectile of mass 100 kg is fired from a point on the surface of the Earth and it travels vertically upwards.

The Earth is modelled as a sphere of radius 6.4×10^6 m and mass 6.0×10^{24} kg.

Assume that all the forces acting on the projectile, except the force of attraction of the Earth, are negligible.

(a) Find the acceleration of the projectile when it is at a distance x m from the **centre** of the Earth. *(3 marks)*

(b) The projectile is fired with speed $u\,\mathrm{m\,s^{-1}}$.

Find the minimum value of u if the projectile is to reach a height of 6×10^6 m above the **surface** of the Earth. *(7 marks)*

3 A satellite, S, orbits the planet Mars in a plane through O, the centre of Mars. The satellite is modelled as a particle of mass m, and Mars is modelled as a stationary sphere. At time t, the length of OS is r and OS is rotating about O with angular speed $\dot{\theta}$. The only force acting on S is directed towards O and has magnitude $km\,\dot{\theta}$, where k is a constant.

When $r = a$, the transverse component of the satellite's velocity is U.

(a) Express $r^2\dot{\theta}$ in terms of a and U. *(4 marks)*

(b) Hence show that $\ddot{r} = \dfrac{aU}{r^2}\left(\dfrac{aU}{r} - k\right)$. *(4 marks)*

(c) Given that a is the minimum distance of the satellite from O during its orbit, deduce that $U \geqslant k$. *(2 marks)*

4 In a chemical process, the quantities of substances X, Y and Z are interdependent. At time t hours, the masses of X, Y and Z are x, y and z grams respectively.

The simultaneous differential equations modelling the process are:

$$\dot{x} = -3x, \quad \dot{y} = 2x - y \quad \text{and} \quad \dot{z} = 3y.$$

(a) Given that $x = 6$ when $t = 0$, show that

$$x = 6\mathrm{e}^{-3t}, \qquad t \geqslant 0.$$ *(3 marks)*

(b) Given that $y = 0$ and $z = 0$ when $t = 0$, find:

(i) y in terms of t; *(6 marks)*

(ii) z in terms of t. *(3 marks)*

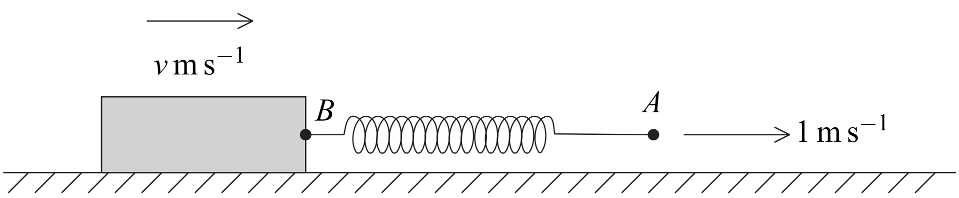
(c) Find the time at which the masses of X and Y are equal. *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

5 A block of mass 0.5 kg moves on a smooth horizontal surface. It is attached to the end B of a light elastic spring AB, where AB is horizontal.

The natural length of the spring is 0.5 m and its stiffness is $5\,\mathrm{N\,m^{-1}}$.

The end A of the spring is pulled horizontally with constant speed of $1\,\mathrm{m\,s^{-1}}$, causing the block to move across the horizontal surface.



At time t seconds, the length of the spring is x metres and the block is moving with speed $v\,\mathrm{m\,s^{-1}}$. The motion of the block is subject to a resistive force of v N.

(a) Write down an expression for the tension in the spring in terms of x. *(2 marks)*

(b) Show that the equation of motion of the block is

$$\frac{\mathrm{d}v}{\mathrm{d}t} + 2v - 10x = -5\,.$$ *(3 marks)*

(c) (i) Explain why $\dot{x} = 1 - v$. *(1 mark)*

(ii) Hence show that

$$\ddot{x} + 2\dot{x} + 10x = 7\,.$$ *(2 marks)*

(d) Find the general solution of this equation. *(6 marks)*

(e) State the limiting value of x. Give a reason for your answer. *(2 marks)*

END OF QUESTIONS