

General Certificate of Education
January 2004
Advanced Level Examination



MATHEMATICS (SPECIFICATION A)
Unit Mechanics 4

MAM4/W

Thursday 29 January 2004 Morning Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAM4/W.
- Answer **all** questions.
- Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise stated.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 John drops a rock of mass m kg from the top of a vertical cliff. At time t seconds after the rock is dropped, it has speed v m s^{-1} and the air resistance to its motion is of magnitude $0.1mv$ newtons.

(a) Show that the differential equation connecting v and t for this motion is $\frac{dv}{dt} = g - 0.1v$.
(2 marks)

(b) Solve this differential equation to find v in terms of g and t .
(5 marks)

(c) Hence show that the speed of the rock cannot exceed 98 m s^{-1} .
(1 mark)

2 [In this question take Newton's gravitational constant to be $6.7 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$.]

A meteorite of mass $5 \times 10^4 \text{ kg}$ is attracted towards the earth. The earth is modelled as a uniform sphere of mass $6 \times 10^{24} \text{ kg}$ and radius $6.4 \times 10^6 \text{ m}$.

Initially, the meteorite is at a distance of $8 \times 10^6 \text{ m}$ from the earth's surface and is moving directly towards the centre of the earth with a speed of 50 m s^{-1} .

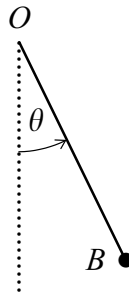
(a) When the distance of the meteorite from the centre of the earth is x metres, show that the magnitude of the acceleration of the meteorite is approximately $\frac{4 \times 10^{14}}{x^2} \text{ m s}^{-2}$.
(4 marks)

(b) Find the speed of the meteorite as it crashes into the earth.
(6 marks)

- 3 A simple pendulum consists of a small particle B suspended from a fixed point O by a light inextensible string of length 0.98 m.

The pendulum makes small oscillations under gravity in a medium which offers a resistance to the motion of B . The magnitude of the resistance is $2mv$, where m is the mass of B and v is the speed of B .

The angular displacement of the pendulum at time t is θ , where θ is measured positively in the direction shown by the arrow in the diagram below.



- (a) Show that when θ is increasing, the tangential equation of motion of B is

$$\ddot{\theta} + 2\dot{\theta} + 10\theta = 0.$$

State the approximation that you have used.

(5 marks)

- (b) Find the general solution of this differential equation.

(4 marks)

- (c) Given that $\theta = \frac{\pi}{20}$ radians and $\dot{\theta} = 0$ when $t = 0$, find the particular solution of the equation.

(5 marks)

- (d) Find the first time, after the motion starts, at which B is momentarily at rest.

(4 marks)

TURN OVER FOR THE NEXT QUESTION

- 4 A sledge of mass 40 kg is sliding across a frozen lake at a speed of 8 m s^{-1} when snow starts to fall vertically on to the sledge. The snow falls on to the sledge at a constant rate of 0.05 kg s^{-1} . The total resistance to the motion of the sledge is assumed to be 5 N.

- (a) By using the impulse/momentum principle, show that the equation of motion of the sledge is

$$(40 + 0.05t) \frac{dv}{dt} + 0.05v = -5,$$

where v is the speed of the sledge t seconds after the snow starts to fall. (5 marks)

- (b) By solving the differential equation in part (a), find v in terms of t . (7 marks)

- 5 A particle P , of mass m , moves in a plane curve. At time t , the polar coordinates of P are (r, θ) relative to a fixed pole O . The path of P is the curve with equation

$$r = \frac{1}{1 + a \cos \theta},$$

where a is a constant.

- (a) Given that P moves under a force directed towards O , show that $r^2 \dot{\theta} = h$, where h is a constant. (2 marks)
- (b) Show that the radial component of velocity of P is $ah \sin \theta$. (4 marks)
- (c) Show that the radial force acting on P is $m \frac{h^2}{r^2}$. (6 marks)

END OF QUESTIONS