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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6300
Specification A

MAM4/W Mechanics 4

Mark Scheme

2005 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

MAM4/W

Q	Solution	Marks	Total	Comments
1(a)(i)	Using "$F = ma$":			
	$9g - 0.01v^2 = 9\dfrac{\mathrm{d}v}{\mathrm{d}t}$	M1A1	2	
(ii)	At terminal velocity, $\dfrac{\mathrm{d}v}{\mathrm{d}t} = 0$	M1		
	$9g - 0.01v^2 = 0$			
	$v^2 = \dfrac{9g}{0.01}$			
	$v = 93.9\ \mathrm{m\,s^{-1}}$ or $42\sqrt{5}\,\mathrm{m\,s^{-1}}$	A1F	2	
(b)	$9g - 0.01v^2 = 9\dfrac{\mathrm{d}v}{\mathrm{d}t}$			
	$9g - 0.01v^2 = 9\dfrac{\mathrm{d}v}{\mathrm{d}x}v$	B1		for using $\dfrac{\mathrm{d}v}{\mathrm{d}t} = v\dfrac{\mathrm{d}v}{\mathrm{d}x}$
	$\int_0^x \mathrm{d}x = 9\int_0^v \dfrac{v\,\mathrm{d}v}{9g - 0.01v^2}$	M1		attempt at integration with correct separation of variables
	$[x]_0^x = \dfrac{-9}{0.02}\left[\ln\left(9g - 0.01v^2\right)\right]_0^v$	A1F		
	$x = -450\left[\ln\left(9g - 0.01v^2\right) - \ln\left(9g\right)\right]$	m1 A1F		correct use of limits or evaluation of constant
	$x = 450\left[\ln\left(9g\right) - \ln\left(9g - 0.01v^2\right)\right]$	A1F	6	
	$x = 450\ \ln\dfrac{9g}{9g - 0.01v^2}$			AG
	Total		**10**	

MAM4/W (cont)

Q	Solution	Marks	Total	Comments
2(a)	By Newton's law of Gravitation:			
	$F = \dfrac{6.7\times10^{-11}\times100\times6.0\times10^{24}}{x^2}$	M1		
	$-\dfrac{6.7\times10^{-11}\times100\times6.0\times10^{24}}{x^2} = 100a$	M1		Using "$F = ma$"
	$a = -\dfrac{4.02\times10^{14}}{x^2}$	A1	3	
(b)	$v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -\dfrac{4.02\times10^{14}}{x^2}$	B1		
	$\int_u^0 v\,\mathrm{d}v = -4.02\times10^{14}\int_{6.4\times10^6}^{12.4\times10^6}\dfrac{\mathrm{d}x}{x^2}$	M1		attempt at integration with correct separation of variables
	$\left[\dfrac{1}{2}v^2\right]_u^0 = \left[4.02\times10^{14}x^{-1}\right]_{6.4\times10^6}^{12.4\times10^6}$	A1F m1		ft on the A1 in (a) use of limits or evaluation of a constant
	$u^2 = 8.04\times10^{14}\left[\dfrac{1}{6.4\times10^6} - \dfrac{1}{12.4\times10^6}\right]$	A2,1		A1 if signs are not correct
	$u = 7800\,\mathrm{m\,s^{-1}}$	A1F	7	AWRT
	Total		**10**	

MAM4/W (cont)

Q	Solution	Marks	Total	Comments
3(a)	No transverse force $\Rightarrow \frac{m}{r}\frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\dot{\theta}\right)=0$	M1A1		
	$r^2\dot{\theta}=C$			
	$aU=C$	M1		
	$r^2\dot{\theta}=aU$	A1	4	
(b)	$m\left(\ddot{r}-r\dot{\theta}^2\right)=-2km\dot{\theta}^2$	M1A1		
	$\ddot{r}=r\dot{\theta}^2-2k\dot{\theta}^2$			
	$\ddot{r}=\frac{r^4\dot{\theta}^2}{r^3}-2k\frac{r^4\dot{\theta}^2}{r^4}$			
	$\ddot{r}=\frac{a^2U^2}{r^3}-2k\frac{a^2U^2}{r^4}$	M1		eliminating $\dot{\theta}$
	$\ddot{r}=\frac{aU}{r^2}\left(\frac{aU}{r}-k\right)$	A1	4	AG
(c)	$r=a\Rightarrow$			
	$\ddot{r}=\frac{U}{a}(U-k)$			
	r is min $\Rightarrow$	M1		
	$\ddot{r}\geq 0\Rightarrow U\geq k$	A1	2	
	Total		**10**	

MAM4/W (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\int_6^x \frac{\mathrm{d}x}{x} = -\int_0^t 3\,\mathrm{d}t$	M1		**Alternative:**
	$\ln x - \ln 6 = -3t$	A1		verify $x = 6$ when $t = 0$ B1 verify $x = 6\mathrm{e}^{-3t}$ satisfies the d.e M1A1
	$x = 6\mathrm{e}^{-3t}$	A1	3	AG
(b)(i)	$\dot{y} = 2x - y$			
	$\dot{y} + y = 12\mathrm{e}^{-3t}$	M1		eliminating x
	$\ddot{y} + \dot{y} = 2\left(-18\mathrm{e}^{-3t}\right)$			**Alternative:**
	$m^2 + m = 0$			Integrating factor is e^t M1
	$m = -1$			$\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^t y\right) = 12\mathrm{e}^{-3t} \times \mathrm{e}^t$ A1F
	CF $y = B\mathrm{e}^{-t}$	M1		$\mathrm{e}^t y = 12\left(-\frac{1}{2}\mathrm{e}^{-2t}\right) + C$ M1A1F
	For PI $y = C\mathrm{e}^{-3t}$	A1F		$t = 0,\ y = 0 \Rightarrow C = 6$ A1F
	$-3C\mathrm{e}^{-3t} + C\mathrm{e}^{-3t} = 12\mathrm{e}^{-3t}$	M1		$\mathrm{e}^t y = -6\mathrm{e}^{-2t} + 6$
	$C = -6$	A1F		$y = 6\mathrm{e}^{-t} - 6\mathrm{e}^{-3t}$ A1F
	GS $y = B\mathrm{e}^{-t} - 6\mathrm{e}^{-3t}$			
	$t = 0,\ y = 0 \ \Rightarrow B = 6$			
	$y = 6\mathrm{e}^{-t} - 6\mathrm{e}^{-3t}$	A1F	6	
(ii)	$\dot{z} = 3\left(6\mathrm{e}^{-t} - 6\mathrm{e}^{-3t}\right)$			
	$z = -18\mathrm{e}^{-t} + 6\mathrm{e}^{-3t} + D$	M1A1F		
	$t = 0,\ z = 0 \Rightarrow D = 12$			
	$z = -18\mathrm{e}^{-t} + 6\mathrm{e}^{-3t} + 12$	A1F	3	
(c)	$6\mathrm{e}^{-3t} = 6\mathrm{e}^{-t} - 6\mathrm{e}^{-3t}$			
	$\mathrm{e}^{-2t} = 0.5$			
	$t = 0.347$ (or 0.35)	M1A1F	2	M1 only for $t = \frac{1}{2}\ln 2$
	Total		**14**	

MAM4/W (cont)

Q	Solution	Marks	Total	Comments
5(a)	$T = 10(x-0.5)$ or $-10(x-0.5)$	M1A1	2	
(b)	$10(x-0.5)-2v = 1\dfrac{dv}{dt}$	M1A1F		Newton's 2nd law
	$\dfrac{dv}{dt}+2v-10x=-5$	A1	3	AG
(c)(i)	The rate of change of the length of the spring = speed of A relative to B $\therefore \dot{x} = 1-v$	E1	1	OE
(ii)	$-\ddot{x}+2(1-\dot{x})-10x=-5$	M1		
	$\ddot{x}+2\dot{x}+10x=7$	A1	2	AG
(d)	Auxiliary equation: $m^2+2m+10=0$	M1		
	$m=-1\pm 3\mathrm{i}$	A1		
	CF $x = e^{-t}(A\sin 3t + B\cos 3t)$	M1		
	PI $x=\alpha$ $\dot{x}=\ddot{x}=0$ $\alpha = \dfrac{7}{10}$	M1 A1F		
	GS $x = e^{-t}(A\sin 3t + B\cos 3t)+\dfrac{7}{10}$	A1F	6	
(e)	$t\to\infty$ $e^{-t}\to 0$ $x\to\dfrac{7}{10}$	M1A1F	2	
	Total		**16**	
	TOTAL		**60**	