

GCE 2004

June Series



Mark Scheme

Mathematics A

Unit MAM4/W

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Dr Michael Cresswell Director General

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for.....	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
✓ or ft or F	follow through from previous	incorrect result
CAO	correct answer only
AWFW	anything which falls within
AWRT	anything which rounds to
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
SF	significant figure(s)
DP	decimal place(s)

Abbreviations used in Marking

MC – x	deducted x marks for mis-copy
MR – x	deducted x marks for mis-read
ISW	ignored subsequent working
BOD	given benefit of doubt
WR	work replaced by candidate
FB	formulae booklet

Application of Mark Scheme

No method shown:

Correct answer without working.....	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

Crossed out work

do not mark unless it has not been replaced

Alternative solution using a correct or partially correct method

award method and accuracy marks as appropriate

MAM4/W

Q	Solutions	Marks	Total	Comments
1(a)	$20e^t - 400v = 800 \frac{dv}{dt}$	M1 A1	3	Newton's 2 nd law
	$\frac{dv}{dt} + \frac{1}{2}v = \frac{1}{40}e^t$	A1		AG
(b)	Integrating factor is $e^{\frac{1}{2}t}$	M1 A1	5	Correct form
	$e^{\frac{1}{2}t} \frac{dv}{dt} + \frac{1}{2}e^{\frac{1}{2}t}v = \frac{1}{40}e^{\frac{3}{2}t}$	M1 A1		
	$e^{\frac{1}{2}t}v = \int \frac{1}{40}e^{\frac{3}{2}t} dt$	A1F		
(c)	$v = 0, t = 0 \Rightarrow C = -\frac{1}{60}$	M1	3	Substituting $t = 6$ At least 3 s.f. required for A1
	$e^{\frac{3}{2}t}v = \frac{1}{60}e^{\frac{3}{2}t} - \frac{1}{60}$	M1		
	$v = 6.72$	A1F		
Total			11	

MAM4/W (Cont)

Q	Solution	Marks	Total	Comments
2(a)	<div style="display: flex; align-items: center; margin-bottom: 5px;"> <div style="border: 1px solid black; padding: 2px 5px; margin-right: 10px;">m</div> <div style="text-align: center;">$\uparrow v$</div> </div> <p>Time t</p> <div style="display: flex; align-items: center; margin-bottom: 5px;"> <div style="border: 1px solid black; padding: 2px 5px; margin-right: 10px;">$m - \delta m$</div> <div style="text-align: center;">$\uparrow v + \delta v$</div> </div> <div style="display: flex; align-items: center; margin-bottom: 5px;"> <div style="border: 1px solid black; padding: 2px 5px; margin-right: 10px;">δm</div> <div style="text-align: center;">$\downarrow u - v$</div> </div> <p>Time $t + \delta t$</p> $(m - \delta m)(v + \delta v) + \delta m(v - u) - mv = -mg \delta t$ $m \frac{\delta v}{\delta t} - \frac{\delta m}{\delta t} \delta v - u \frac{\delta m}{\delta t} = -mg$ $m \frac{dv}{dt} - u \frac{dm}{dt} = -mg$ $\frac{dv}{dt} = \frac{u}{m} \frac{dm}{dt} - g$ $\frac{dv}{dt} = \frac{u\alpha}{M - \alpha t} - g$	<p>B1</p> <p>M1 A2,1 m1</p> <p>m1</p> <p>A1</p>	<p>7</p>	<p>p.i.</p> <p>Attempt to use the principle -1 ee First order</p> <p>AG</p>
(b)	<p>The motor will stop when all fuel is burnt.</p> $\frac{2}{3}M = \alpha t$ $t = \frac{2M}{3\alpha}$	<p>B1</p>	<p>1</p>	<p>o.e.</p> <p>AG</p>
(c)	$\int_0^v dv = \int_0^t \left(\frac{u\alpha}{M - \alpha t} - g \right) dt$ $v = -u \ln(M - \alpha t) - gt + u \ln M$ $v = -u \ln \left(M - \alpha \frac{2M}{3\alpha} \right) - g \frac{2M}{3\alpha} + u \ln M$ $v = -u \ln \frac{1}{3}M + u \ln M - \frac{2gM}{3\alpha}$ $v = u \ln 3 - \frac{2gM}{3\alpha}$	<p>M1 A2,1 m1 A1F</p> <p>A1F</p>	<p>6</p>	<p>-1 ee</p>
Total			14	

MAM4/W (Cont)

Q	Solution	Marks	Total	Comments
3(a)	$\dot{\vartheta} = \frac{2\pi}{5} \text{ rad s}^{-1}$	B1	1	Or equivalent
(b)	Transverse force $F = m(r\ddot{\vartheta} + 2\dot{r}\dot{\vartheta})$ $= 60(2 \times 2 \times \frac{2\pi}{5})$ $= 96\pi \text{ N } (= 302 \text{ N})$	M1 A1	2	
(c)	Max. Central Force $F = -m(\ddot{r} - r\dot{\vartheta}^2)$ $= -60(-4(\frac{2\pi}{5})^2)$ $= \frac{192}{5}\pi^2 \text{ N } (=379\text{N})$	M1 A1	2	
Total			5	

MAM4/W (Cont)

Q	Solution	Marks	Total	Comments
4(a)	$12\dot{x} = y$ $\dot{y} = 2y - 9x$ $\ddot{y} = 2\dot{y} - 9\dot{x}$ $\ddot{y} - 2\dot{y} + 9\left(\frac{y}{12}\right) = 0$ $4\ddot{y} - 8\dot{y} + 3y = 0$	M1 ml A1	3	Substitution for \dot{x}
(b)	$4m^2 - 8m + 3 = 0$ $m = \frac{1}{2}, \frac{3}{2}$ $y = Ae^{\frac{1}{2}t} + Be^{\frac{3}{2}t}$ $y = n, x = 500, t = 0$ $n = A + B$ $\dot{y} = \frac{1}{2}Ae^{\frac{1}{2}t} + \frac{3}{2}Be^{\frac{3}{2}t}$ $2y - 9x = \frac{1}{2}Ae^{\frac{1}{2}t} + \frac{3}{2}Be^{\frac{3}{2}t}$ $2n - 4500 = \frac{1}{2}A + \frac{3}{2}B$ $B = \frac{3}{2}n - 4500, A = 4500 - \frac{1}{2}n$	M1 A1 M1 ml ml ml A1F A1F	8	Attempt at solving aux
(c)	$n = 2800$ $y = 3100e^{\frac{1}{2}t} - 300e^{\frac{3}{2}t}$ $y = 0 \Rightarrow e^t = \frac{3100}{300}$ $t = \ln \frac{3100}{300}$ $t = 2.34$	M1 ml A1F	3	For setting $y = 0$ & $n = 2800$ AWRT
Total			14	

MAM4/W (Cont)

Q	Solution	Marks	Total	Comments
5(a)		B1	1	
(b)	$T = 4.5\left(\frac{1}{2}\sin 2t - x\right)$ $T = 0.5\ddot{x}$ $0.5\ddot{x} = 4.5\left(\frac{1}{2}\sin 2t - x\right)$ $\ddot{x} + 9x = \frac{9}{2}\sin 2t$	M1 A1 M1		Newton's 2 nd law
(c)	<p>Auxiliary eqⁿ. $m^2 + 9 = 0$</p> $m = \pm 3i$ <p>C.F. $x = A\cos 3t + B\sin 3t$</p> <p>For P.I. $x = p\cos 2t + q\sin 2t$</p> $\dot{x} = -2p\sin 2t + 2q\cos 2t$ $\ddot{x} = -4p\cos 2t - 4q\sin 2t$ $5p\cos 2t + 5q\sin 2t = \frac{9}{2}\sin 2t$ $p = 0, q = \frac{9}{10}$ <p>G.S. is</p> $x = A\cos 3t + B\sin 3t + \frac{9}{10}\sin 2t$ $x = \dot{x} = 0, t = 0$ $\therefore A = 0$ $\dot{x} = 3B\cos 3t + \frac{9}{5}\cos 2t$ $\therefore B = -\frac{3}{5}$ $x = -\frac{3}{5}\sin 3t + \frac{9}{10}\sin 2t$	M1 A1 B1 m1 M1 A1F A1F A1F	4	AG For both p and q
			9	

MAM4/W (Cont)

Q	Solution	Marks	Total	Comments
5 (d)	$\dot{x} = -\frac{9}{5}\cos 3t + \frac{9}{5}\cos 2t$ $= -\frac{9}{5}\cos \frac{6\pi}{5} + \frac{9}{5}\cos \frac{4\pi}{5}$ $= 0$	M1 A1	 2	
	Total		16	
	Total		60	