General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) MAM3
Unit Mechanics 3

Friday 24 June 2005 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAM3.
- Answer **all** questions.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$ unless otherwise stated.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

1 A heavy horizontal disc is free to rotate about a fixed vertical axis through its centre. A child stands at the centre of the disc.

Initially the child stands with his arms at his side. The disc is rotating at an angular speed of $3\ \text{rad}\ \text{s}^{-1}$.

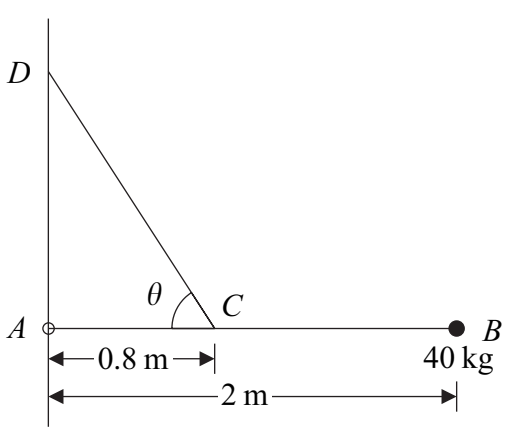
(a) The moment of inertia, about the axis, of the disc and child combined in the initial position is $1.4\ \text{kg}\ \text{m}^2$.

Find the angular momentum of the disc and child. *(2 marks)*

(b) The child now raises his arms slowly to the horizontal position. When they are fully extended, the angular speed is observed to have reduced to $2.5\ \text{rad}\ \text{s}^{-1}$.

Find the moment of inertia, about the axis, of the child and disc combined in this new position. *(2 marks)*

2 A uniform horizontal beam AB has mass 50 kg and length 2 metres. The end A of the beam is attached by a smooth hinge to a vertical wall. The beam is maintained in a horizontal position by a light inextensible rope, which is attached to the beam at the point C, 0.8 metres from A, and to the wall at the point D, directly above A. The rope is at an angle θ to the horizontal. A mass of 40 kg is attached to the beam at B.



(a) Draw a diagram to show the forces acting on the beam. *(2 marks)*

(b) The breaking strength of the rope is 2000 N. Show that angle θ must be at least 53°, correct to the nearest degree. *(6 marks)*

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3 Forces $\begin{bmatrix} 2 \\ -8 \end{bmatrix}$, $\begin{bmatrix} a \\ b \end{bmatrix}$ and $\begin{bmatrix} -3 \\ -4 \end{bmatrix}$ act at points $(0, 0)$, $(3, 2)$ and $(4, -2)$ respectively. The three forces are equivalent to a single force $\begin{bmatrix} 3 \\ -6 \end{bmatrix}$.

(a) Find the values of a and b. *(3 marks)*

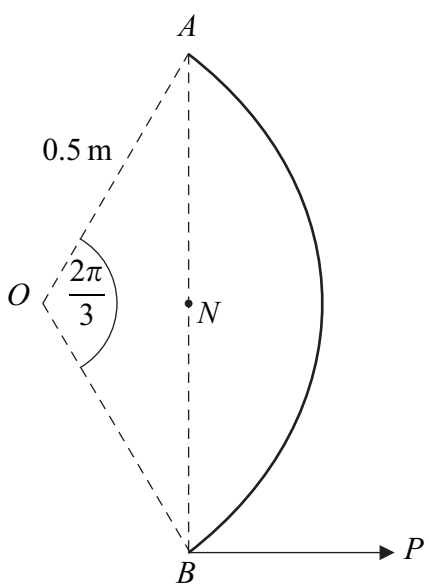
(b) (i) Find the magnitude of the resultant moment of the forces about the origin. *(4 marks)*

(ii) State the sense of this moment. *(1 mark)*

(c) The line of action of the single equivalent force cuts the y-axis at point $(0, d)$. Find the value of d. *(2 marks)*

4 The diagram below shows a uniform wire AB, bent into the form of an arc of a circle subtending an angle of $\frac{2\pi}{3}$ radians at the centre O. The arc is of radius 0.5 m and N is the mid-point of the chord AB.

The wire is smoothly suspended at point A and is in equilibrium with point B vertically below point A, under the action of a horizontal force P acting at B.



(a) Draw a diagram showing the lines of action of:

(i) the weight of the wire; *(1 mark)*

(ii) the force on the wire at A. *(1 mark)*

(b) Show that the centre of mass of the wire is at a distance of approximately 0.163 m from N. *(3 marks)*

(c) Given that the mass of the wire is 0.03 kg, find the value of P. *(3 marks)*

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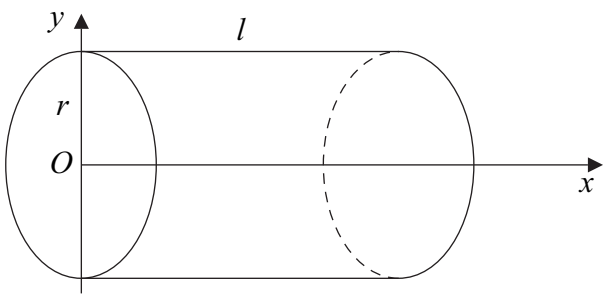
5 (a) A uniform solid cylinder of mass M has radius r and length l.



You may assume that the moment of inertia of a uniform disc of mass m and radius r about an axis along a diameter is $\frac{mr^2}{4}$.

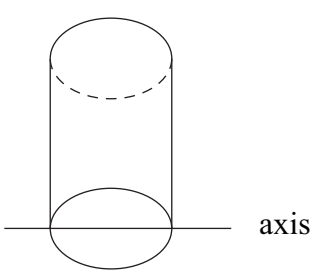
Show, using integration, that the moment of inertia of the cylinder about an axis lying along the diameter of a plane face is $M\left(\frac{r^2}{4}+\frac{l^2}{3}\right)$. *(7 marks)*

(b) A uniform solid cylinder of mass M, radius a and length $3a$ is free to rotate about a horizontal axis lying along the diameter of a plane face.

(i) Show that the moment of inertia of the cylinder about this axis is $\frac{13Ma^2}{4}$.

(1 mark)

(ii) Initially, the cylinder is held with its centre of mass directly above the axis, as shown in the diagram below.



The cylinder is gently disturbed from its initial position, and in the subsequent motion has rotated through an angle θ at time t.

Find $\dot{\theta}^2$ in terms of a, g and θ. *(4 marks)*

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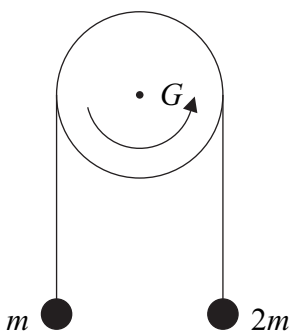
6 A cylindrical drum, of radius r, rotates about a horizontal axle lying along its axis. The moment of inertia of the drum about this axis is $\frac{3mr^2}{2}$. A light inextensible cord lies over the drum and connects particles of masses m and $2m$. The system is released from rest at time $t = 0$. During the motion a constant frictional couple G acts on the drum and the cord does **not** slip on the drum. At time t the drum has turned through an angle θ.



(a) (i) Write down the equation of motion for each of the two particles. *(3 marks)*

(ii) Write down the equation of rotational motion for the drum. *(3 marks)*

(iii) Hence show that $\ddot{\theta} = \frac{2(mgr - G)}{9mr^2}$. *(5 marks)*

(b) When the drum has turned through an angle α, the cord is completely detached from the drum without any impulse. The drum continues to rotate under the action of the frictional couple G only.

(i) Show that, at the moment the cord is detached, $\dot{\theta}^2 = \frac{4(mgr - G)\alpha}{9mr^2}$. *(2 marks)*

(ii) The drum then turns through a further angle β before being brought to rest by the frictional couple. Find G in terms of α, β, m, g and r. *(5 marks)*

END OF QUESTIONS

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