General Certificate of Education
January 2005
Advanced Level Examination

**MATHEMATICS (SPECIFICATION A)** **MAD2**
**Unit Discrete 2**

Thursday 27 January 2005 Afternoon Session

**In addition to this paper you will require:**
- an 8-page answer book;
- an insert for use in Questions 2 and 4 (enclosed);
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

**Instructions**
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAD2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used, including the insert for use in Questions 2 and 4, to the back of your answer book before handing it to the invigilator.

**Information**
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.
- Further copies of the insert for use in Questions 2 and 4 are available on request.

**Advice**
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

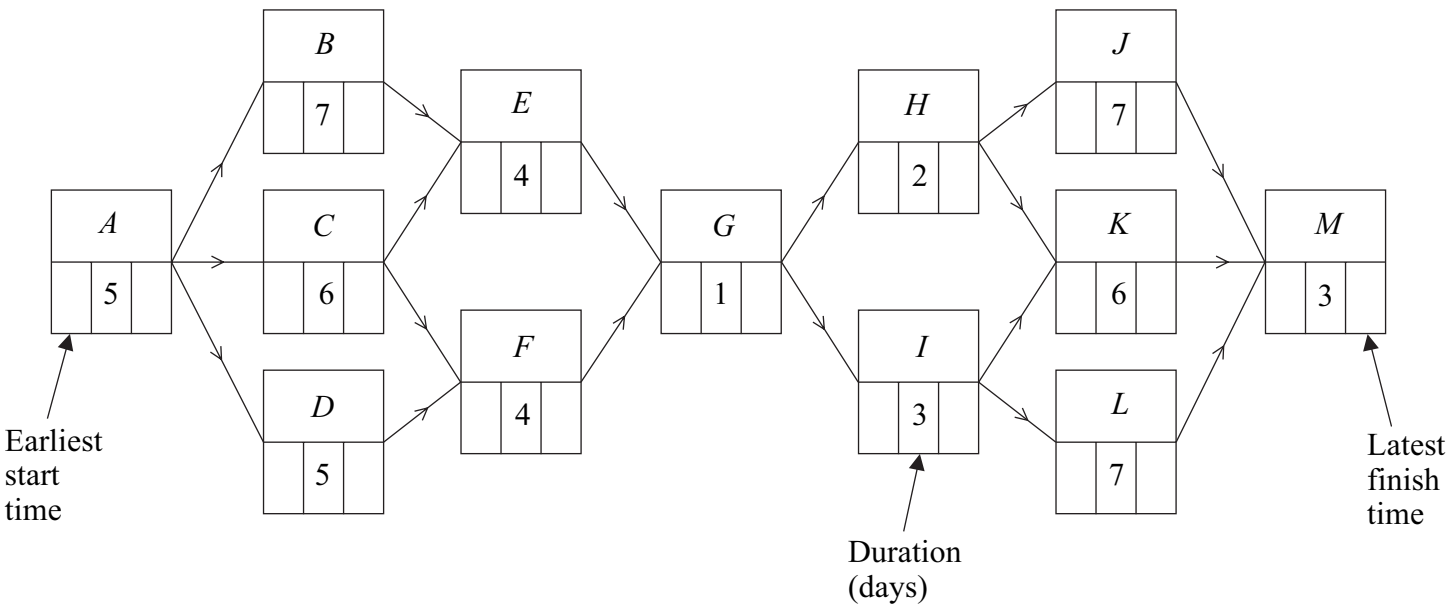
**1** An office manager has four secretaries available and four tasks to be completed. Each secretary is to be assigned a task. The time, in minutes, required by each secretary to complete each task is given in the table.

| | **Task 1** | **Task 2** | **Task 3** | **Task 4** |
|---|---|---|---|---|
| **A** | 23 | 27 | 21 | 26 |
| **B** | 28 | 28 | 26 | 27 |
| **C** | 24 | 25 | 23 | 24 |
| **D** | 26 | 24 | 23 | 21 |

Use the Hungarian algorithm to obtain the allocation of secretaries to tasks that minimises the total time. State this minimum time. *(6 marks)*

**2** [**Figure 1**, printed on the insert, is provided for use in answering this question.]

The following diagram shows an activity network for a major project.




(a) On **Figure 1**:

(i) find the earliest start time for each activity; *(2 marks)*

(ii) find the latest finish time for each activity. *(2 marks)*

(b) Identify the critical path. *(1 mark)*

(c) Write down the activity that has a float time of more than 1 day. *(1 mark)*

(d) Given that activity *C* overruns by 3 days, find the overrun time for the whole project. *(2 marks)*

(e) Given that activities *C* and *F* overrun by 3 days each, find the overrun time for the whole project. *(3 marks)*

**3** Cara is to fly from Manchester to Palermo. The journey has three legs.

Leg 1 is from Manchester to Stansted, Gatwick or Luton.
Leg 2 is from one of these airports to Turin, Rome or Venice.
Leg 3 is from Turin, Rome or Venice to Palermo.

Her travelling time consists of flying time and changeover time.

**Figure 2** shows travelling times between airports and **Figure 3** shows changeover times. All times are in minutes.

| | **Manchester** (*M*) | **Stansted** (*S*) | **Gatwick** (*G*) | **Luton** (*L*) | **Turin** (*T*) | **Rome** (*R*) | **Venice** (*V*) | **Palermo** (*P*) |
|---|---|---|---|---|---|---|---|---|
| **Manchester** (*M*) | — | 50 | 45 | 40 | — | — | — | — |
| **Stansted** (*S*) | 50 | — | — | — | 52 | 60 | 48 | — |
| **Gatwick** (*G*) | 45 | — | — | — | 55 | 65 | 60 | — |
| **Luton** (*L*) | 40 | — | — | — | 70 | 75 | 65 | — |
| **Turin** (*T*) | — | 52 | 55 | 70 | — | — | — | 55 |
| **Rome** (*R*) | — | 60 | 65 | 75 | — | — | — | 40 |
| **Venice** (*V*) | — | 48 | 60 | 65 | — | — | — | 60 |
| **Palermo** (*P*) | — | — | — | — | 55 | 40 | 60 | — |

**Figure 2**

| | **Turin** (*T*) | **Rome** (*R*) | **Venice** (*V*) | **Palermo** (*P*) |
|---|---|---|---|---|
| **Stansted** (*S*) | 75 | 70 | 75 | — |
| **Gatwick** (*G*) | 75 | 70 | 70 | — |
| **Luton** (*L*) | 70 | 64 | 70 | — |
| **Turin** (*T*) | — | — | — | 70 |
| **Rome** (*R*) | — | — | — | 80 |
| **Venice** (*V*) | — | — | — | 65 |

**Figure 3**

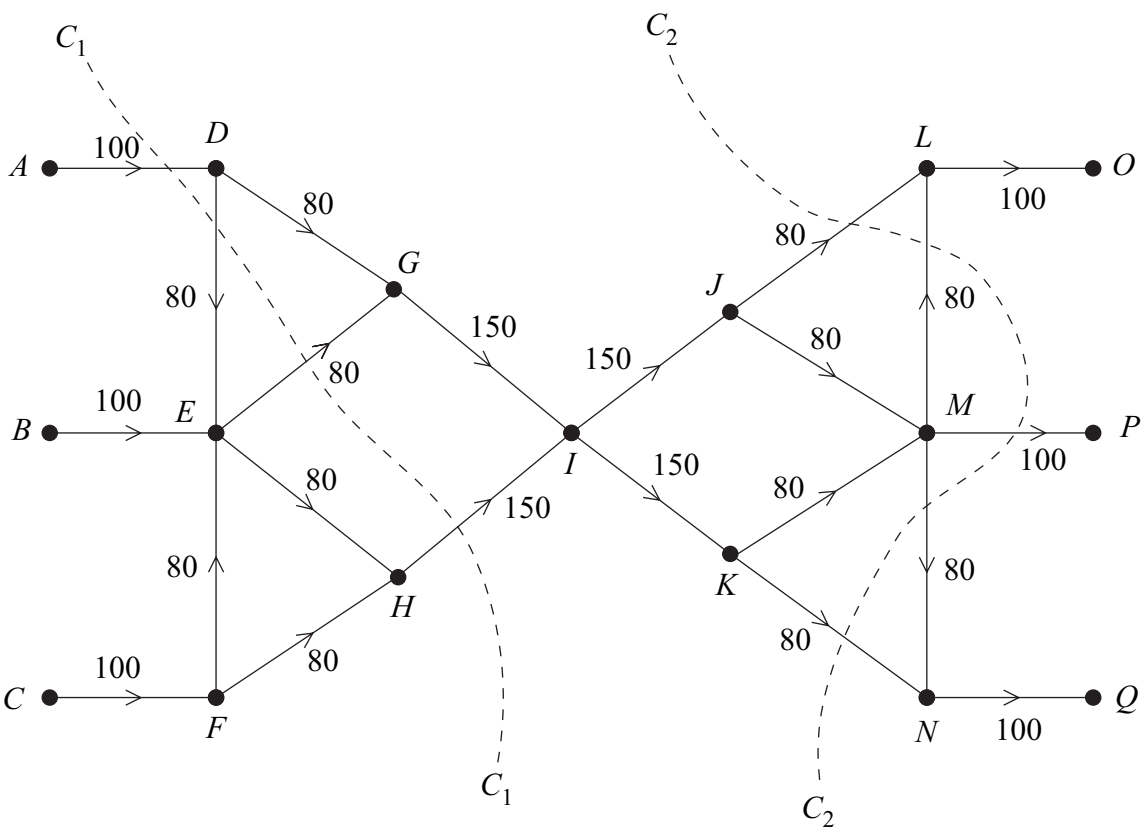
(a) Show that the total time taken for Cara to travel from Manchester to Palermo via Gatwick and Rome is 300 minutes. *(2 marks)*

(b) By drawing a network diagram, or otherwise, use dynamic programming to find the minimum time for Cara to travel from Manchester to Palermo. State the corresponding route. *(9 marks)*

**TURN OVER FOR THE NEXT QUESTION**

**4** [**Figure 4**, printed on the insert, is provided for use in answering this question.]

The following diagram shows the numbers of pupils who can pass along various corridors of a school building in one minute in the event of a fire drill. The classroom areas are $A$, $B$ and $C$. The fire exits are $O$, $P$ and $Q$.




(a) Find the values of the cuts $C_1$ and $C_2$. *(2 marks)*

(b) On **Figure 4**, show a flow of 300 pupils per minute through the network. *(3 marks)*

(c) (i) On a certain day there is a restriction at vertex $I$ of 120 pupils per minute.

Find the maximum flow on this day. *(1 mark)*

(ii) On another day the junction at $I$ is clear but there is a restriction at vertex $H$ of 120 pupils per minute.

Find the maximum flow on this day. *(2 marks)*

**5** (a) Display the following linear programming problem in a Simplex tableau.

$$\begin{aligned} \text{Maximise} \quad & P = 2x + 3y \\ \text{subject to} \quad & 2x + y \leqslant 20 \\ & x + 2y \leqslant 20 \\ & 5x + 4y \leqslant 60 \end{aligned}$$

*(2 marks)*

(b) Solve the problem using the Simplex algorithm, giving your answers as exact fractions. *(9 marks)*

**6** Two people, $A$ and $B$, play a zero-sum game. The game is represented by the following pay-off matrix for $A$.

| | | $B$ | | |
|---|---|---|---|---|
| | **Strategy** | **I** | **II** | **III** |
| $A$ | **I** | 4 | −1 | 2 |
| | **II** | 2 | 3 | 1 |
| | **III** | 1 | −2 | 1 |

(a) Explain why it will never be optimal for $A$ to adopt strategy III. *(1 mark)*

(b) (i) Find the optimal mixed strategy for $A$. *(7 marks)*

(ii) Show that the value of the game is $\frac{7}{5}$. *(1 mark)*

(c) By considering optimal mixed strategies for $B$, show that player $B$ should play strategy II with a probability of $\frac{1}{5}$. *(4 marks)*

**END OF QUESTIONS**

| Surname | | Other Names | |
|---|---|---|---|
| Centre Number | | | Candidate Number | | | | |
| Candidate Signature | | | | | | | | | |

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Insert for use in answering Questions 2 and 4.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

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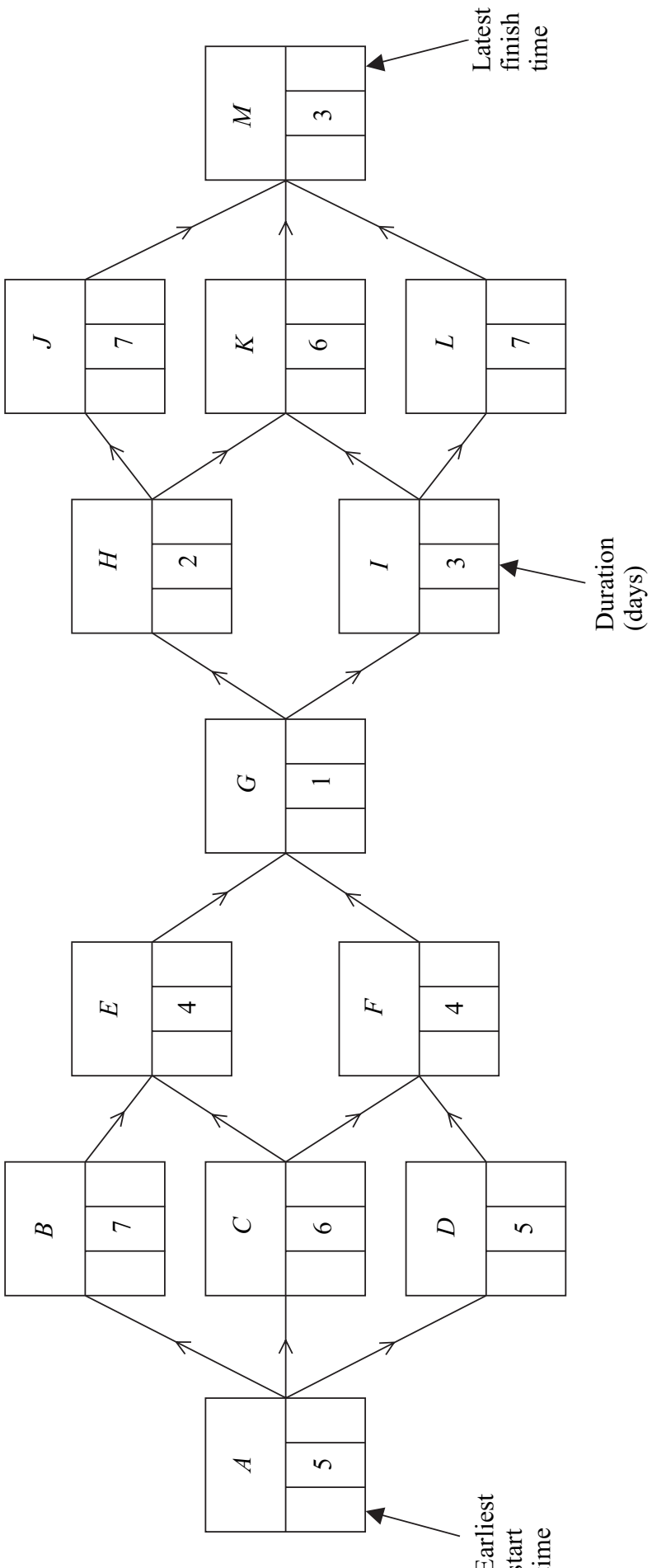


**Figure 1 (for Question 2)**

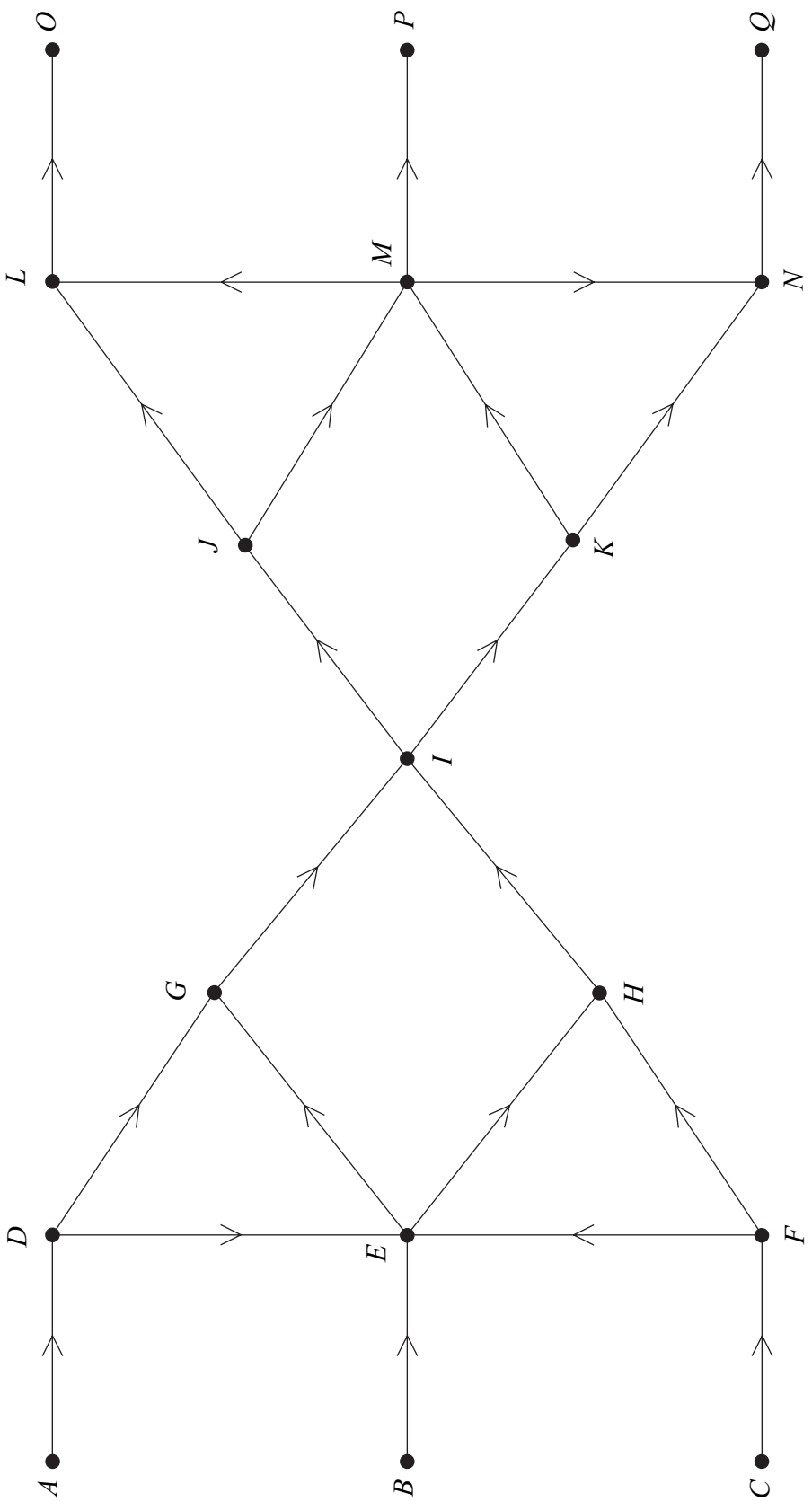


**Figure 4 (for Question 4)**

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