General Certificate of Education
January 2004
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAD2**
Unit Discrete 2

Friday 23 January 2004 Morning Session

In addition to this paper you will require:
• an 8-page answer book;
• an insert for use in Questions 2, 3 and 4 (enclosed);
• one sheet of graph paper for use in Question 2;
• the AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAD2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used, including the insert for use in Questions 2, 3 and 4, to the back of the answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.
- Further copies of the insert for use in Questions 2, 3 and 4 are available on request.
- Further sheets of graph paper are available on request.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Four lecturers can each deliver any one of the four modules A, B, C and D of a course.

The lecturers have been rated by previous students as to how well they deliver each module. They are rated on a scale of 1 to 10, with 1 being the top rating. The ratings are shown in the table below.

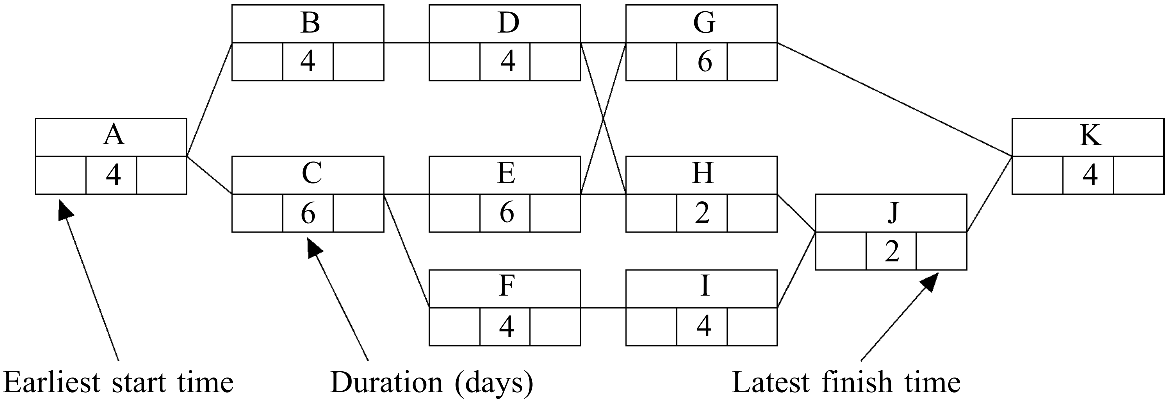
	A	***B***	***C***	***D***
Edwina	1	3	2	1
Hilary	8	10	9	10
Michael	7	9	8	7
Rick	3	2	4	1

Each lecturer is to be assigned to deliver one module.

Use the Hungarian algorithm to obtain the allocation of lecturers to modules that has the minimum total rating. State this minimum. *(6 marks)*

2 [**Figure 1**, printed on the insert, is provided for use in answering this question.]

The diagram shows an activity network for a building project.



(a) On **Figure 1**:

(i) find the earliest start time for each activity; *(2 marks)*

(ii) find the latest finish time for each activity. *(2 marks)*

(b) Identify the critical path and state the minimum time for completion of the project. *(1 mark)*

(c) Given that one activity overruns by four days, but the completion time is unaffected, list the activities which could overrun. *(1 mark)*

(d) Each activity requires one worker in order for the activity to be completed in the stated time.

Draw a resource histogram for the project. *(3 marks)*

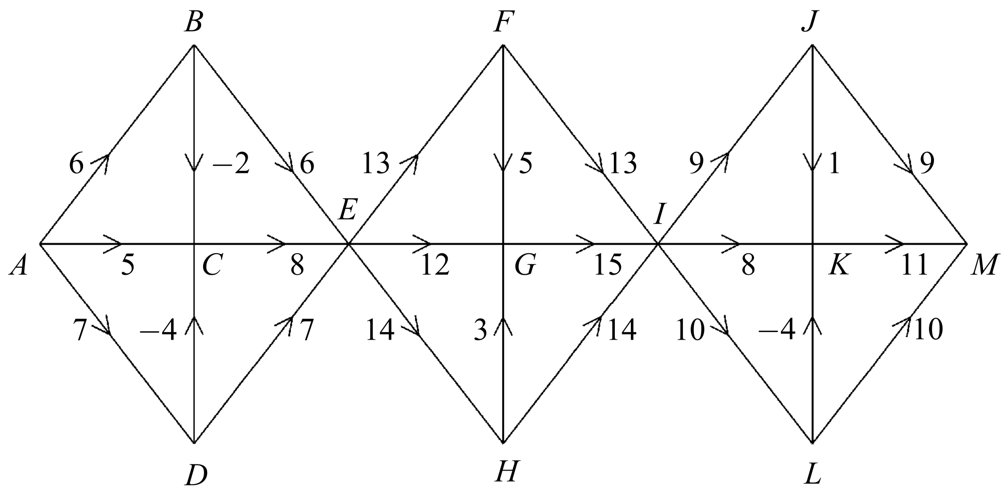
(e) Given that there are only two workers available, each being able to complete any of the activities they undertake in the stated time:

(i) state the new minimum time to complete the project, giving a reason for your answer; *(2 marks)*

(ii) state how the activities could be divided between the two workers to achieve this time. *(1 mark)*

3 [**Figure 2**, printed on the insert, is provided for use in answering this question.]

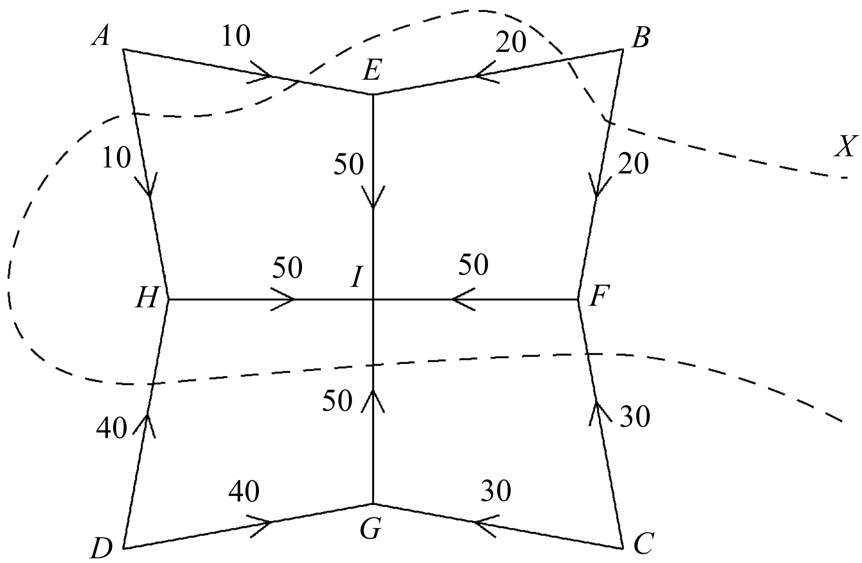
The following network shows 13 vertices. The number on each arc is the cost of a journey between two vertices.



Use dynamic programming on **Figure 2** to find the minimum cost of a route from A to M. State the route corresponding to this minimum cost. *(7 marks)*

4 [**Figures 3** and **4**, printed on the insert, are provided for use in answering this question.]

The following diagram represents a network of pipes used in a water system in a garden. The number on each arc represents the maximum volume of water, in cubic centimetres, that can flow along the corresponding pipe in one second.



(a) State the vertices that represent the sources. *(1 mark)*

(b) State the vertex that represents the sink. *(1 mark)*

(c) Find the value of the cut X. *(2 marks)*

(d) (i) On **Figure 3**, starting from a position of zero flow, use flow augmentation to find the maximum flow. *(4 marks)*

(ii) State why this flow is maximum. *(1 mark)*

(e) On a particular day there is a restriction at vertex E which allows a maximum flow through E of 20.

On **Figure 4**, find, by inspection, the maximum flow through the network on this day. *(2 marks)*

5 (a) A student is solving a linear programming question using the Simplex algorithm. The student obtains the following Simplex tableau.

P	x	y	z	r	s	
5	0	0	8	1	9	12
0	5	0	-2	1	-1	2
0	0	15	12	-1	6	3

(i) Explain why this tableau is optimal. *(1 mark)*

(ii) Solve this tableau, stating the values of x, y, z and P. *(2 marks)*

(b) Solve the following linear programming problem using the Simplex algorithm.

Maximise $P = 3x + 6y + 2z$,

subject to $3x + 4y + 2z \leqslant 2$,

$x + 3y + 2z \leqslant 1$,

and $x \geqslant 0, \quad y \geqslant 0, \quad z \geqslant 0$. *(9 marks)*

6 (a) Afzal and Bill play a zero-sum game. The game is represented by the following pay-off matrix for Afzal.

		Bill I	Bill II	Bill III	Bill IV
Afzal	I	3	-2	1	2
	II	3	5	2	6
	III	0	2	-3	3

(i) Show that this game has a stable solution. *(3 marks)*

(ii) List any saddle points. *(1 mark)*

(b) Colin and David play a zero-sum game. The game is represented by the following pay-off matrix for Colin.

		David I	David II
Colin	I	$x+2$	$x-1$
	II	3	5

Given that the optimal mixed strategy makes the value of the game $\frac{19}{5}$, find the value of x. *(8 marks)*

END OF QUESTIONS